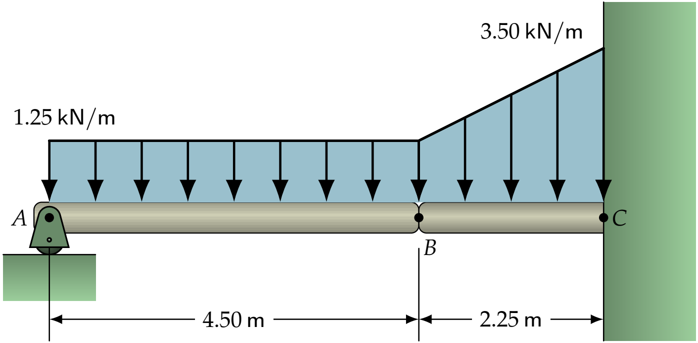
Complex Frames — Step by Step Examples

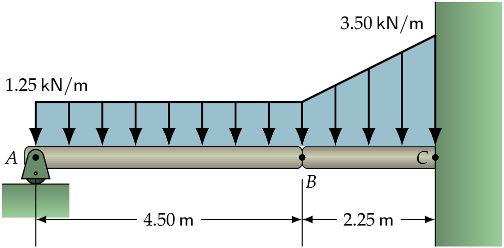
Engineering Statics

Last revision on December 27, 2025



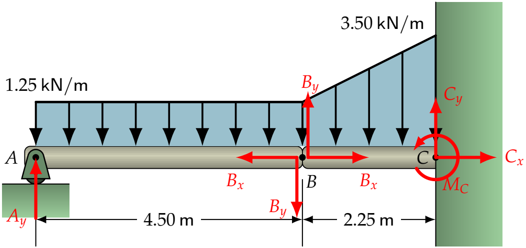
Complex Frames: Example 1

There is a roller at A, a pinned connection at B and a fixed connection at C. Determine the reactions at A and C.



Example 1: Our Method

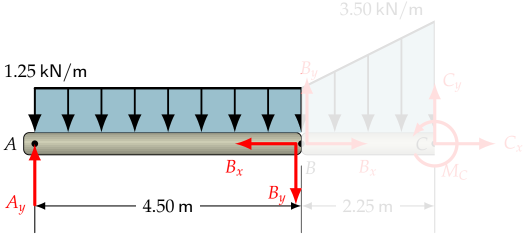
- How many unknowns are there in this problem?



Example 1: Our Method

- How many unknowns are there in this problem?

  There are six unknowns: A_y, B_x, B_y, C_x, C_y and M_C. And two members, AB and BC, so the problem is statically determinant.

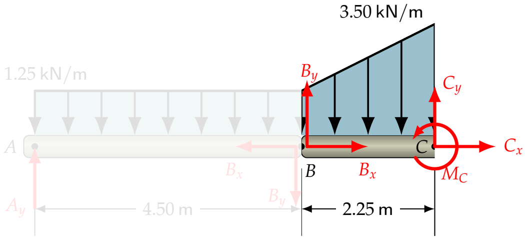


Example 1: Our Method

- How many unknowns are there in this problem?

  There are six unknowns: A_y, B_x, B_y, C_x, C_y and M_C. And two members, AB and BC, so the problem is statically determinant.

- Member AB has three unknowns and is solvable. We start there.



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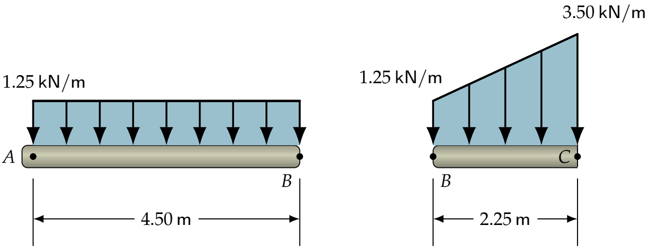
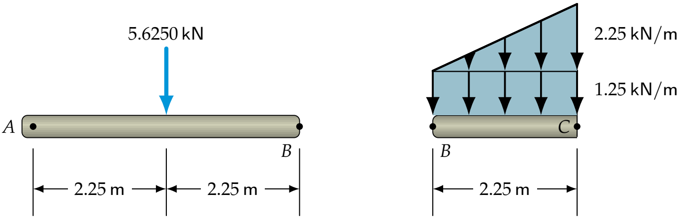
  There are six unknowns: A_y, B_x, B_y, C_x, C_y and M_C. And two members, AB and BC, so the problem is statically determinant.
- Member AB has three unknowns and is solvable. We start there.
- Now that B_x and B_y are known, member BC is solvable.



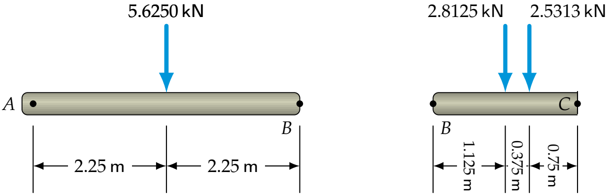
Example 1: The Solution

- Separate the members for more convenient analysis.



Example 1: The Solution

- Separate the members for more convenient analysis.
- Resolve the distributed loads.



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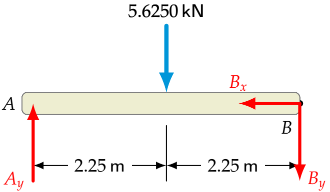
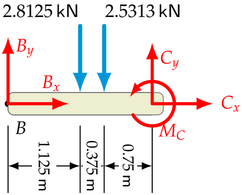
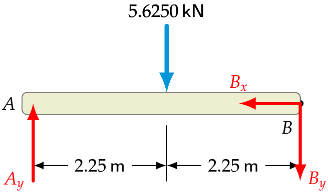
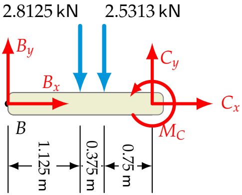




Example 1: The Solution

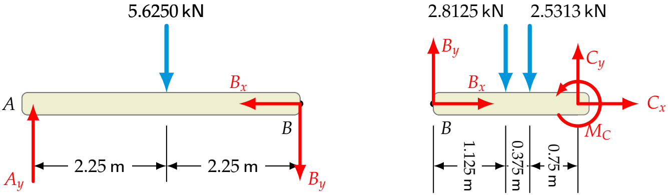
- Separate the members for more convenient analysis.
- Resolve the distributed loads.
- Draw free body diagrams.





Example 1: The Solution

- Separate the members for more convenient analysis.
- Resolve the distributed loads.
- Draw free body diagrams.
- Analyze member AB.



Example 1: Member AB

$$\Sigma M_B = 5.6250\,\text{kN}\cdot(2.25\,\text{m}) - A_y\cdot(4.50\,\text{m}) = 0$$

$$\Rightarrow A_y = 2.8125\,\text{kN}$$

$$\Sigma F_x = -B_x = 0$$

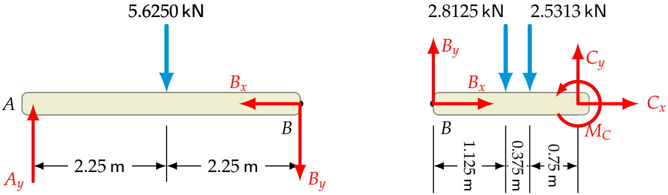
$$\Sigma F_y = 2.8125\,\text{kN} - 5.6250\,\text{kN} - B_y = 0$$

$$\Rightarrow B_y = -2.8125\,\text{kN}$$

$$A_y = 2.8125\,\text{kN}$$

$$B_x = 0$$

$$B_y = -2.8125\,\text{kN}$$



Example 1: Member BC

$$\Sigma M_C = M_C + 2.5313\,\text{kN}\cdot(0.750\,\text{m}) + 2.8125\,\text{kN}\cdot(1.1250\,\text{m}) - (-2.8125\,\text{kN})\cdot(2.25\,\text{m}) = 0$$

$$\Rightarrow M_C = -11.391\,\text{kN}\cdot\text{m}$$

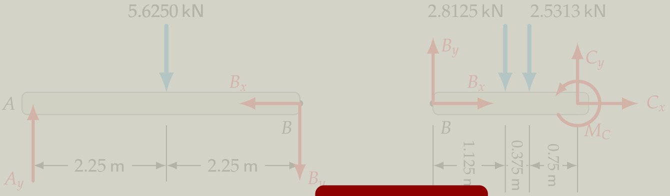
$$\Sigma F_x = C_x + B_x = 0$$

$$\Rightarrow C_x = 0$$

$$\Sigma F_y = C_y + -2.8125\,\text{kN} - 2.8125\,\text{kN} - 2.5313\,\text{kN} = 0$$

$$\Rightarrow C_y = 8.1563\,\text{kN}$$

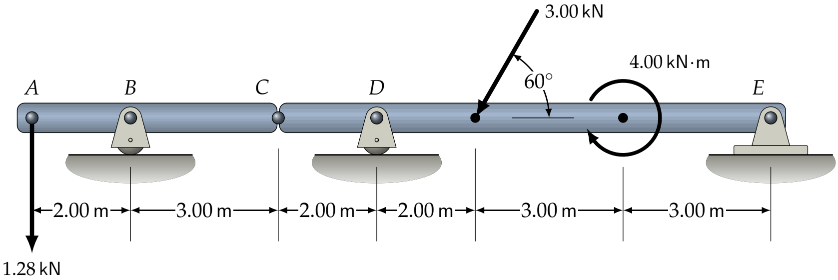
$$A_y = 2.8125\,\text{kN}$$
$$B_x = 0$$
$$B_y = -2.8125\,\text{kN}$$
$$M_C = -11.391\,\text{kN}\cdot\text{m}$$
$$C_x = 0$$
$$C_y = 8.1563\,\text{kN}$$



The Answers

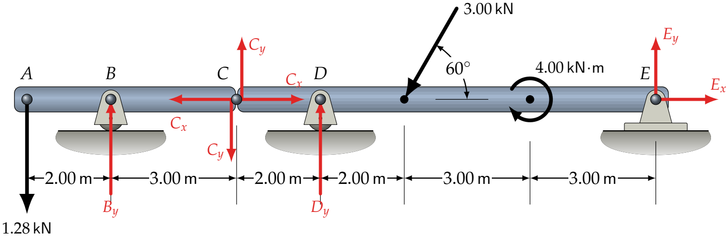
$$R_A = 2.81\,\text{kN at } 90^\circ$$
$$R_C = 8.52\,\text{kN at } 90^\circ$$
$$M_C = -11.4\,\text{kN·m}$$

$$M_C = -11.391\,\text{kN·m}$$
$$C_x = 0$$
$$C_y = 8.1563\,\text{kN}$$



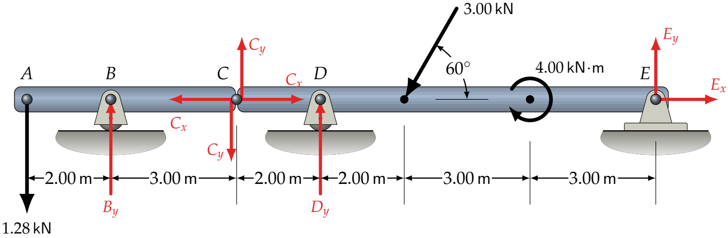
Complex Frames: Example 2

There are frictionless rollers at B and D and pinned connections at C and E. Determine the reactions at B, D and E.



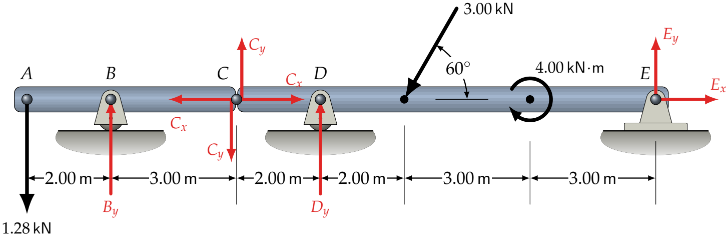
Example 2: Our Method

- There are six unknowns (B_y, C_x, C_y, D_y, E_x and E_y) and two members (AC and CE) so the problem is statically determinant.



Example 2: Our Method

- There are six unknowns (B_y, C_x, C_y, D_y, E_x and E_y) and two members (AC and CE) so the problem is statically determinant.
- This example employs a similar method to the previous one: We first check for a member that has only three unknowns and, if such a member exists, solve for those three unknowns.



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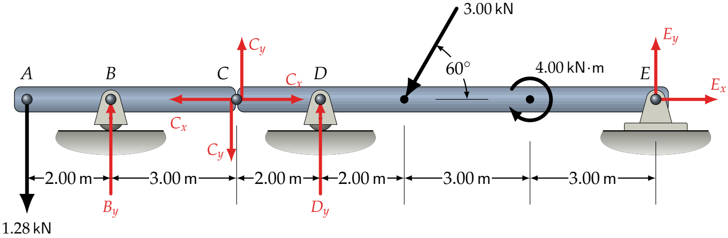
- This example employs a similar method to the previous one: We first check for a member that has only three unknowns and, if such a member exists, solve for those three unknowns.
- Member AC has three unknowns and is solvable. We start there.



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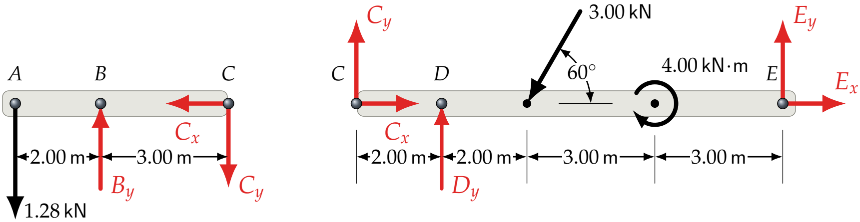
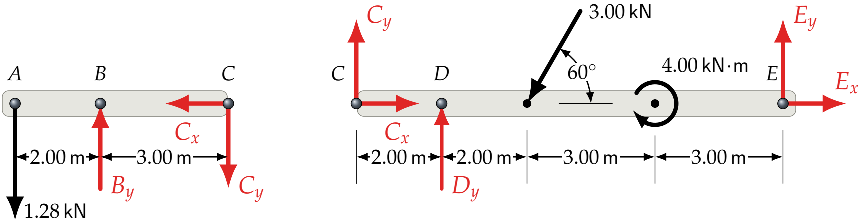
- There are six unknowns (B_y, C_x, C_y, D_y, E_x and E_y) and two members (AC and CE) so the problem is statically determinant.
- This example employs a similar method to the previous one: We first check for a member that has only three unknowns and, if such a member exists, solve for those three unknowns.
- Member AC has three unknowns and is solvable. We start there.
- Analysis of AC yields C_x and C_y. Once these are known, member CE is solvable.



Example 2: The Solution

- Draw FBDs for the two members ABC and CDE.

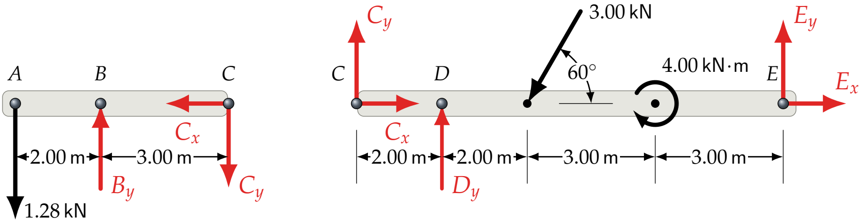


Example 2: The Solution

- Draw FBDs for the two members *ABC* and *CDE*.
- Solve member *AC*.

$$B_y = 2.1333\,\text{kN}$$
$$C_x = 0, C_y = 0.85330\,\text{kN}$$

$$\Sigma M_C = 1.28\,\text{kN}\cdot(5.00\,\text{m}) - B_y\cdot(3.00\,\text{m}) = 0$$
$$\Rightarrow B_y = 2.1333\,\text{kN}$$
$$\Sigma F_x = C_x = 0$$
$$\Sigma F_y = -1.28\,\text{kN} + B_y - C_y = 0$$
$$\Rightarrow C_y = 0.85330\,\text{kN}$$



Example 2: The Solution

- Draw FBDs for the two members *ABC* and *CDE*.
- Solve member *AC*.
- Solve member *CE*.

$$B_y = 2.1333\,\text{kN}$$
$$C_x = 0, C_y = 0.85330\,\text{kN}$$
$$D_y = 0.38193\,\text{kN}$$
$$E_x = 1.5000\,\text{kN}, E_y = 1.1170\,\text{kN}$$
$$R_E = 2.0266\,\text{kN at } 29.74^\circ$$

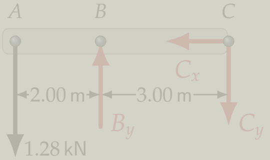
$$\begin{aligned}\Sigma M_E &= -4.00\,\text{kN}\cdot\text{m} \\ &\qquad + (3.00\,\text{kN}\cdot\sin 60^\circ)\cdot(6.00\,\text{m}) \\ &\qquad - D_y\cdot(8.00\,\text{m}) - C_y\cdot(10.0\,\text{m}) = 0 \\ \Rightarrow D_y &= 0.38193\,\text{kN}\end{aligned}$$

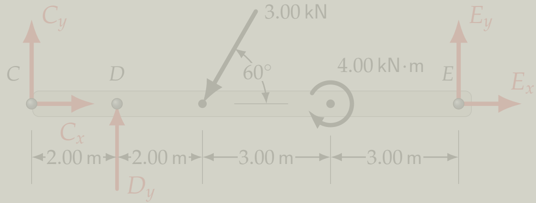
$$\begin{aligned}\Sigma F_x &= E_x - 3.00\,\text{kN}\cdot\cos 60^\circ + C_x = 0 \\ \Rightarrow E_x &= 1.5000\,\text{kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= E_y + D_y + C_y - 3.00\,\text{kN}\cdot\sin 60^\circ = 0 \\ \Rightarrow E_y &= 1.3628\,\text{kN}\end{aligned}$$

$$\begin{aligned}R_E &= \sqrt{(1.5000\,\text{kN})^2 + (1.3628\,\text{kN})^2} \\ &= 2.0266\,\text{kN}\end{aligned}$$

$$R_E\theta = \tan^{-1}\left(\frac{1.3628}{1.5000}\right) = 42.256^\circ$$





Example 2: The Solution

- Draw FBDs for the two members ABC and CDE.
- Solve member AC.
- Solve member CE.

$$B_y = 2.1333\,\text{kN}$$
$$C_x = 0, C_y = 0.85330\,\text{kN}$$
$$D_y = 0.38193\,\text{kN}$$
$$E_x = 1.5000\,\text{kN}, E_y = 1.1170\,\text{kN}$$
$$R_E = 2.0266\,\text{kN at } 29.74^\circ$$

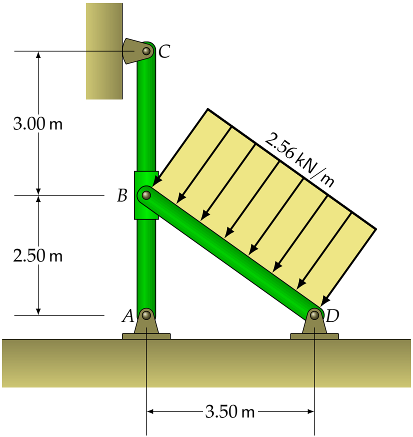
$$\Sigma M_E = -4.00\,\text{kN}\cdot\text{m} + (3.00\,\text{kN}\cdot\sin 60^\circ)\cdot(6.00\,\text{m})$$

$$= 2.0266\,\text{kN}$$

$$R_E\theta = \tan^{-1}\left(\frac{1.3628}{1.5000}\right) = 42.256^\circ$$

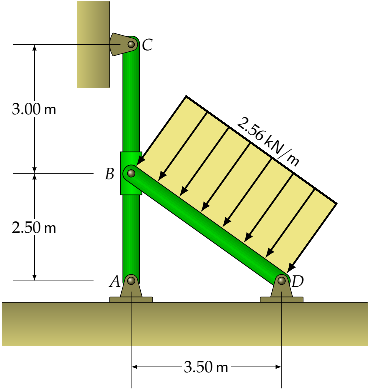
The Answers

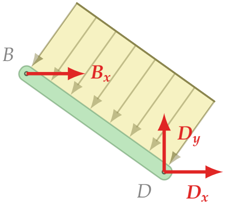
$$R_B = 2.13\,\text{kN at } 90^\circ$$
$$R_D = 0.382\,\text{kN at } 90^\circ$$
$$R_E = 2.03\,\text{kN at } 29.7^\circ \text{ (ccw from +ve } x\text{-axis)}$$



Complex Frames: Example 3

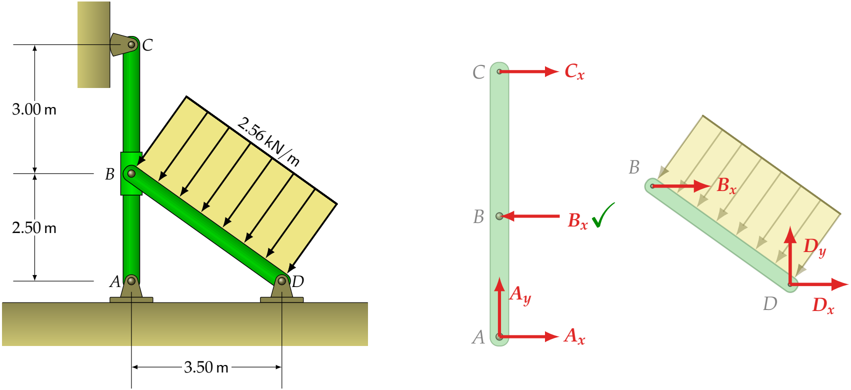
There is a smooth collar at B, a rocker at C and pinned connections at A and D. Determine the force that the collar at B exerts on member BD, and the reactions at A and D.





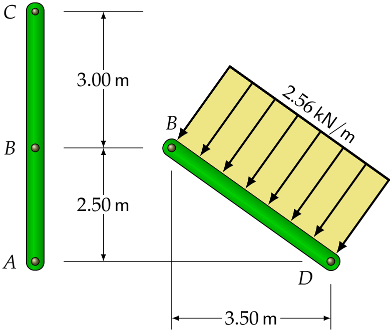
Example 3: Our Method

- As the collar at B is frictionless and has no vertical force component, BD has three unknowns (B_x, D_x and D_y). We solve this first.



Example 3: Our Method

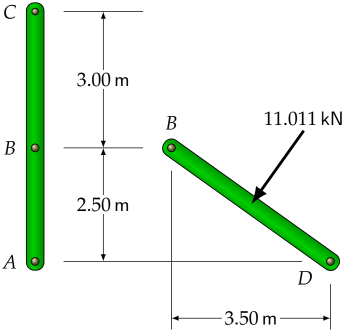
- As the collar at B is frictionless and has no vertical force component, BD has three unknowns (B_x, D_x and D_y). We solve this first.
- Knowing the force B_x, acting at B, we can now solve member AC.



Example 3: Member BD

- Resolve the distributed load.

$$|BD| = \sqrt{(2.50\,\text{m})^2 + (3.50\,\text{m})^2} = 4.3012\,\text{m}$$

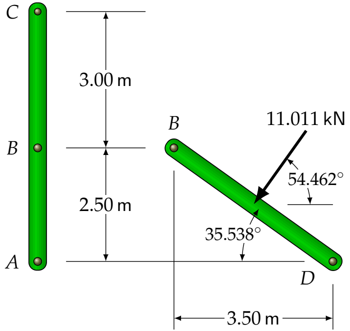


Example 3: Member BD

- Resolve the distributed load.

$$|BD| = \sqrt{(2.50\,\mathrm{m})^2 + (3.50\,\mathrm{m})^2} = 4.3012\,\mathrm{m}$$

$$W = 2.56\,\mathrm{kN/m} \cdot 4.3012\,\mathrm{m} = 11.011\,\mathrm{kN}$$



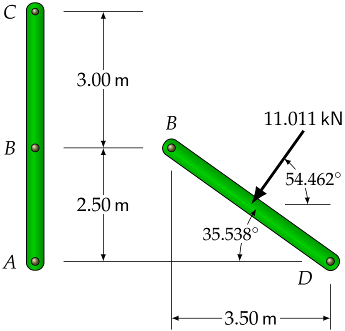
Example 3: Member BD

- Resolve the distributed load.
- Resolve this distributed force load into x- and y-components.

$$|BD| = \sqrt{(2.50\,\text{m})^2 + (3.50\,\text{m})^2} = 4.3012\,\text{m}$$

$$W = 2.56\,\text{kN/m} \cdot 4.3012\,\text{m} = 11.011\,\text{kN}$$

$$\angle BDA = \tan^{-1}\left[\frac{2.50\,\text{m}}{3.50\,\text{m}}\right] = 35.538^\circ$$



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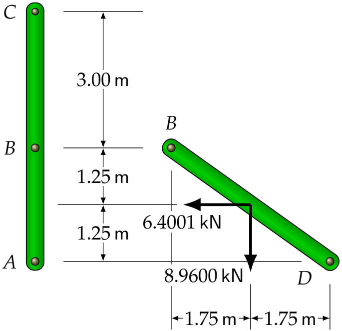
$$\angle BDA = \tan^{-1}\left[\frac{2.50\,\mathrm{m}}{3.50\,\mathrm{m}}\right] = 35.538^\circ$$

$$W_x = 11.011\,\mathrm{kN} \cdot \cos 54.462^\circ = 6.4001\,\mathrm{kN}$$
$$W_y = 11.011\,\mathrm{kN} \cdot \sin 54.462^\circ = 8.9600\,\mathrm{kN}$$



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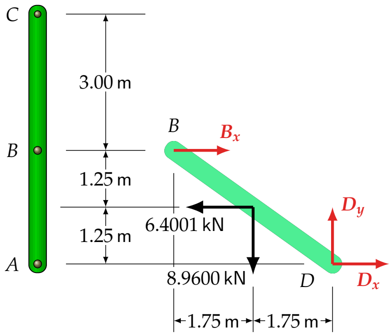

Example 3: Member BD

- Resolve the distributed load.
- Resolve this distributed force load into x- and y-components.
- Complete the FBD for BD.

$$|BD| = \sqrt{(2.50\,\text{m})^2 + (3.50\,\text{m})^2} = 4.3012\,\text{m}$$

$$W = 2.56\,\text{kN/m} \cdot 4.3012\,\text{m} = 11.011\,\text{kN}$$

$$\angle BDA = \tan^{-1}\left[\frac{2.50\,\text{m}}{3.50\,\text{m}}\right] = 35.538^\circ$$

$$W_x = 11.011\,\text{kN} \cdot \cos 54.462^\circ = 6.4001\,\text{kN}$$
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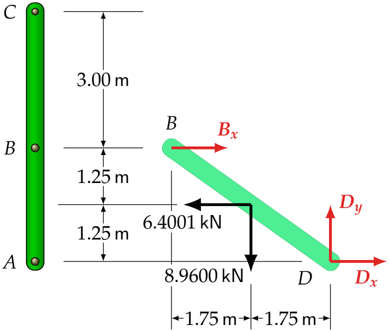


Example 3: Member BD

- Resolve the distributed load.
- Resolve this distributed force load into x- and y-components.
- Complete the FBD for BD.
- Sum the moments about D.

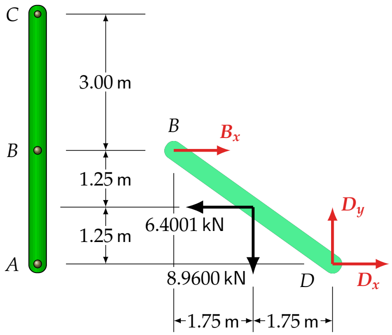
$$|BD| = \sqrt{(2.50\,\text{m})^2 + (3.50\,\text{m})^2} = 4.3012\,\text{m}$$

$$W = 2.56\,\text{kN/m} \cdot 4.3012\,\text{m} = 11.011\,\text{kN}$$

$$\angle BDA = \tan^{-1}\left[\frac{2.50\,\text{m}}{3.50\,\text{m}}\right] = 35.538^\circ$$

$$\begin{aligned} W_x &= 11.011\,\text{kN} \cdot \cos 54.462^\circ = 6.4001\,\text{kN} \\ W_y &= 11.011\,\text{kN} \cdot \sin 54.462^\circ = 8.9600\,\text{kN} \end{aligned}$$

$$\begin{aligned} \Sigma M_D &= 8.9600\,\text{kN} \cdot 1.75\,\text{m} \\ &\quad + 6.4001\,\text{kN} \cdot 1.25\,\text{m} \\ &\quad - B_x \cdot 2.50\,\text{m} = 0 \\ \Rightarrow B_x &= 9.4721\,\text{kN} \end{aligned}$$



Example 3: Member BD

- Resolve the distributed load.
- Resolve this distributed force load into x- and y-components.
- Complete the FBD for BD.
- Sum the moments about D.
- Sum the x- and y-components to find the reaction R_D at D.

$$|BD| = \sqrt{(2.50\,\mathrm{m})^2 + (3.50\,\mathrm{m})^2} = 4.3012\,\mathrm{m}$$

$$W = 2.56\,\mathrm{kN/m} \cdot 4.3012\,\mathrm{m} = 11.011\,\mathrm{kN}$$

$$\angle BDA = \tan^{-1}\left[\frac{2.50\,\mathrm{m}}{3.50\,\mathrm{m}}\right] = 35.538^\circ$$

$$\begin{aligned} W_x &= 11.011\,\mathrm{kN} \cdot \cos 54.462^\circ = 6.4001\,\mathrm{kN} \\ W_y &= 11.011\,\mathrm{kN} \cdot \sin 54.462^\circ = 8.9600\,\mathrm{kN} \end{aligned}$$

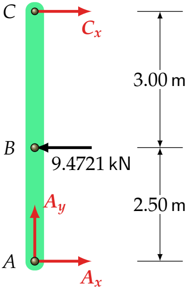
$$\begin{aligned} \Sigma M_D &= 8.9600\,\mathrm{kN} \cdot 1.75\,\mathrm{m} \\ &\quad + 6.4001\,\mathrm{kN} \cdot 1.25\,\mathrm{m} \\ &\quad - B_x \cdot 2.50\,\mathrm{m} = 0 \\ \Rightarrow B_x &= 9.4721\,\mathrm{kN} \end{aligned}$$

$$\begin{aligned} \Sigma F_x &= 9.4721\,\mathrm{kN} - 6.4001\,\mathrm{kN} + D_x = 0 \\ \Rightarrow D_x &= -3.0720\,\mathrm{kN} \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= D_y - 8.9600\,\mathrm{kN} = 0 \\ \Rightarrow D_y &= 8.9600\,\mathrm{kN} \end{aligned}$$

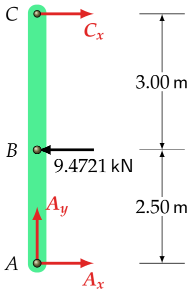
$$\begin{aligned} R_D &= \sqrt{(-3.0720\,\mathrm{kN})^2 + (8.9600\,\mathrm{kN})^2} \\ &= 9.4720\,\mathrm{kN} \end{aligned}$$

$$R_D\theta = 180^\circ - \tan^{-1}\left(\frac{8.9600}{3.0720}\right) = 108.92^\circ$$



Example 3: Member ABC

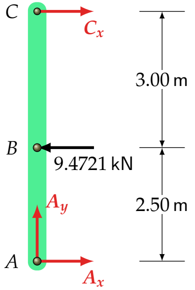
- Complete the FBD for ABC.



Example 3: Member ABC

- Complete the FBD for ABC.
- Sum the moments about A to find the reaction at C.

$$\Sigma M_A = 9.4721\,\text{kN} \cdot 2.50\,\text{m} - C_x \cdot 5.50\,\text{m} = 0$$
$$\Rightarrow C_x = 4.3055\,\text{kN}$$



Example 3: Member ABC

- Complete the FBD for ABC.
- Sum the moments about A to find the reaction at C.
- Sum the x- and y-components to find the reaction R_A at A.

$$\Sigma M_A = 9.4721\,\text{kN} \cdot 2.50\,\text{m} - C_x \cdot 5.50\,\text{m} = 0$$
$$\Rightarrow C_x = 4.3055\,\text{kN}$$

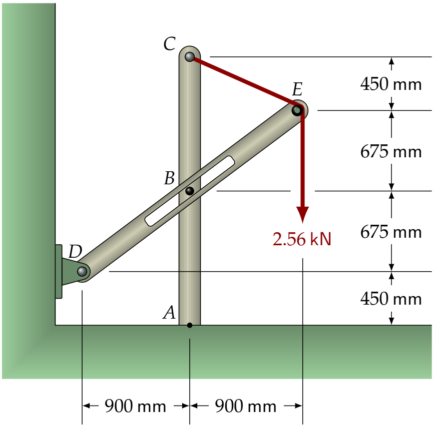
$$\Sigma F_x = 4.3055\,\text{kN} - 9.4721\,\text{kN} + A_x = 0$$
$$\Rightarrow A_x = 5.1666\,\text{kN}$$

$$\Sigma F_y = A_y = 0$$

$$\Rightarrow R_A = \sqrt{(5.1666\,\text{kN})^2 + (0\,\text{kN})^2} = 5.1666\,\text{kN}$$
$$R_A\theta = 0^\circ$$

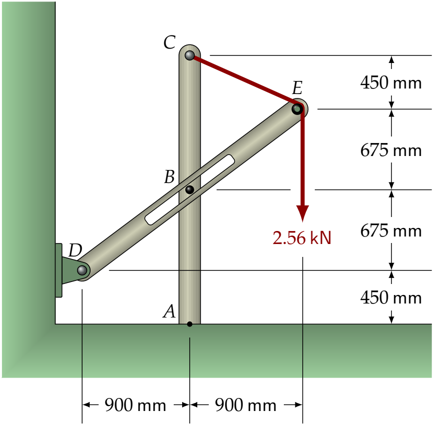
The Answers

$$R_A = 5.17\,\text{kN at } 0^\circ \text{ (in the direction of the +ve } x\text{-axis)}$$
$$B_x = 9.47\,\text{kN at } 0^\circ \text{ (in the direction of the +ve } x\text{-axis)}$$
$$R_D = 9.47\,\text{kN at } 108.92^\circ \text{ (ccw from +ve } x\text{-axis)}$$



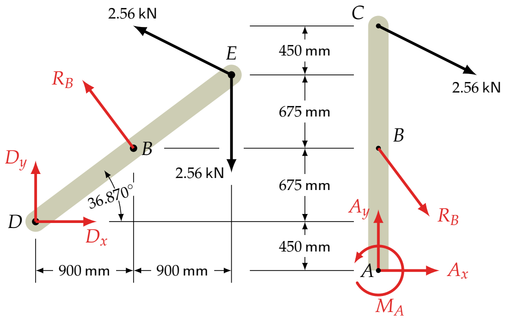
Complex Frames: Example 4

There are smooth pegs B and E, and a pinned connection at D. A is fixed connection. Determine the reactions at A and D.



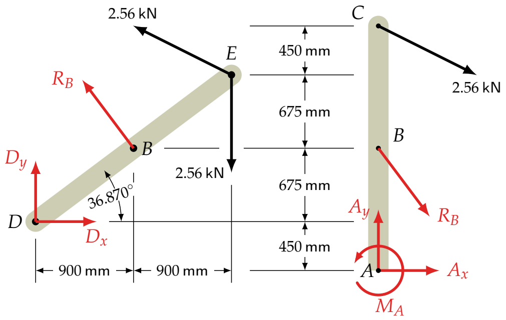
Example 4: Our Method

- Looking at member DBE, there are three unknowns (D_x, D_y and R_B). We start there. and it is solvable. We start there.
- Once we know R_B, there are three remaining unknowns (A_x, A_y and M_C), which we can then find.



Example 4: The Solution

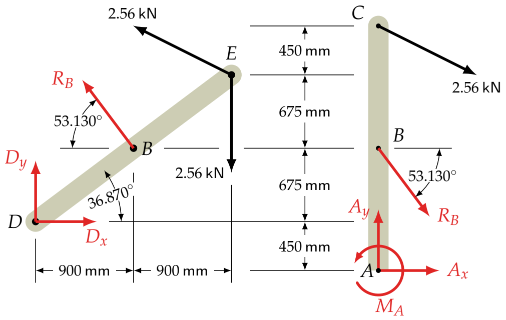
1. Draw separate FBD diagrams.



Example 4: The Solution

1. Draw separate FBD diagrams.
2. To fully specify the FBDs, there are some calculations required:
   2.1 The angle of DBE to the horizontal

$$DBE_\theta = \tan^{-1}\left[\frac{675}{900}\right] = 36.870^\circ$$

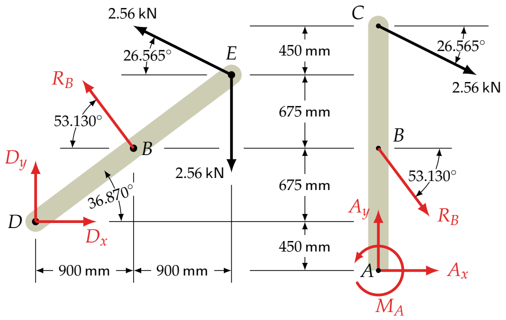


Example 4: The Solution

1. Draw separate FBD diagrams.
2. To fully specify the FBDs, there are some calculations required:
   2.1 The angle of DBE to the horizontal
   2.2 The angle of R_B to the horizontal

$$DBE_\theta = \tan^{-1}\left[\frac{675}{900}\right] = 36.870^\circ$$

$$RB_\theta = 90^\circ - 36.870^\circ = 53.130^\circ$$



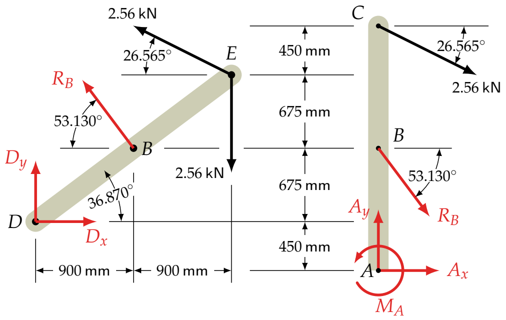
Example 4: The Solution

1. Draw separate FBD diagrams.
2. To fully specify the FBDs, there are some calculations required:
   2.1 The angle of DBE to the horizontal
   2.2 The angle of R_B to the horizontal
   2.3 The angle of cable CE, to the horizontal

$$DBE_\theta = \tan^{-1}\left[\frac{675}{900}\right] = 36.870^\circ$$

$$RB_\theta = 90^\circ - 36.870^\circ = 53.130^\circ$$

$$CE_\theta = \tan^{-1}\left[\frac{450}{900}\right] = 26.565^\circ$$



Example 4: The Solution

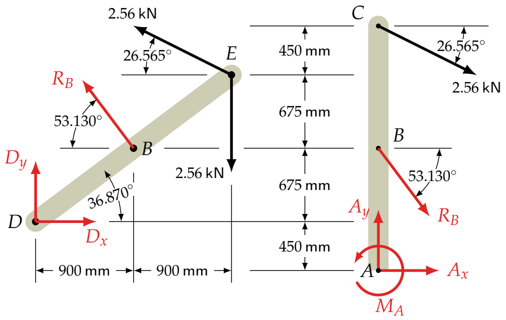
1. Draw separate FBD diagrams.
2. To fully specify the FBDs, there are some calculations required:
   2.1 The angle of DBE to the horizontal
   2.2 The angle of R_B to the horizontal
   2.3 The angle of cable CE, to the horizontal
   2.4 The length from B to D.

$$DBE_\theta = \tan^{-1}\left[\frac{675}{900}\right] = 36.870^\circ$$

$$RB_\theta = 90^\circ - 36.870^\circ = 53.130^\circ$$

$$CE_\theta = \tan^{-1}\left[\frac{450}{900}\right] = 26.565^\circ$$

$$|BD| = \sqrt{(900\,\text{mm})^2 + (675\,\text{mm})^2}$$
$$= 1125\,\text{mm}$$



Example 4: The Solution

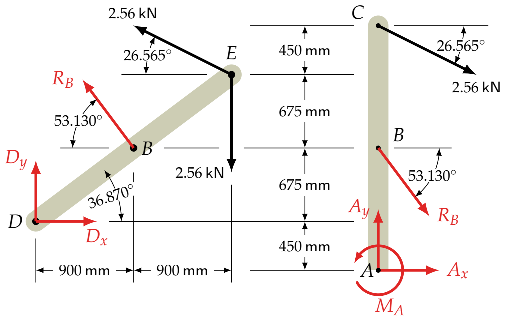
1. Draw separate FBD diagrams.
2. To fully specify the FBDs, there are some calculations required:
   2.1 The angle of DBE to the horizontal
   2.2 The angle of R_B to the horizontal
   2.3 The angle of cable CE, to the horizontal
   2.4 The length from B to D.
3. Now we can proceed with analysis of member DBE.

$$DBE_\theta = \tan^{-1}\left[\frac{675}{900}\right] = 36.870^\circ$$

$$RB_\theta = 90^\circ - 36.870^\circ = 53.130^\circ$$

$$CE_\theta = \tan^{-1}\left[\frac{450}{900}\right] = 26.565^\circ$$

$$|BD| = \sqrt{(900\,\text{mm})^2 + (675\,\text{mm})^2}$$
$$= 1125\,\text{mm}$$

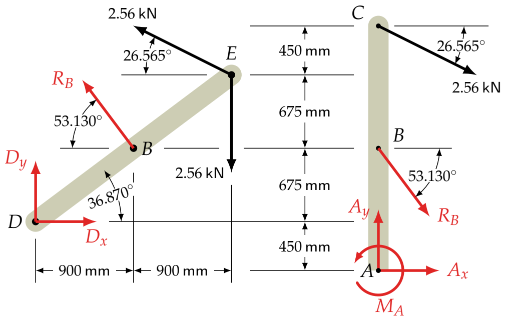


$R_B = -0.48347\,\text{kN}$

Example 4: The Solution

4. Sum the moments about D to find R_B.

$$\begin{aligned}\Sigma M_D = R_B &\times 1125\,\text{mm} \\ &+ 2.56\,\text{kN} \cdot \sin 26.565^\circ \times 1800\,\text{mm} \\ &+ 2.56\,\text{kN} \cdot \cos 26.565^\circ \times 1350\,\text{mm} \\ &- 2.56\,\text{kN} \times 1800\,\text{mm} = 0 \\ \Rightarrow R_B &= -0.48347\,\text{kN}\end{aligned}$$



$$
\begin{aligned}
D_x &= 1.9997\,\text{kN} \\
D_y &= 1.8019\,\text{kN} \\
R_B &= -0.48347\,\text{kN}
\end{aligned}
$$

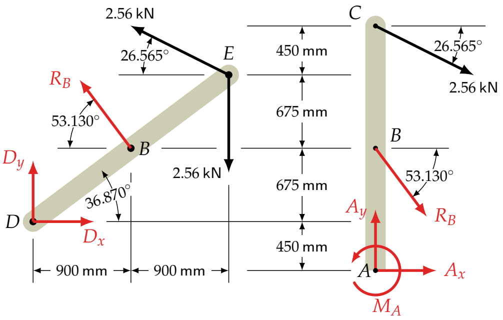
Example 4: The Solution

4. Sum the moments about D to find R_B.
5. Sum the x- and y-components of the forces acting on DBE to find R_D.

$$
\begin{aligned}
\Sigma M_D &= R_B \times 1125\,\text{mm} \\
&\quad + 2.56\,\text{kN} \cdot \sin 26.565^\circ \times 1800\,\text{mm} \\
&\quad + 2.56\,\text{kN} \cdot \cos 26.565^\circ \times 1350\,\text{mm} \\
&\quad - 2.56\,\text{kN} \times 1800\,\text{mm} = 0 \\
\Rightarrow R_B &= -0.48347\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= D_x - (-0.48347\,\text{kN}) \cos 53.130^\circ \\
&\quad - 2.56\,\text{kN} \cdot \cos 26.565^\circ = 0 \\
\Rightarrow D_x &= 1.9997\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_y &= D_y + (-0.48347\,\text{kN}) \sin 53.130^\circ \\
&\quad + 2.56\,\text{kN} \cdot \sin 26.565^\circ - 2.56\,\text{kN} = 0 \\
\Rightarrow D_y &= 1.8019\,\text{kN}
\end{aligned}
$$



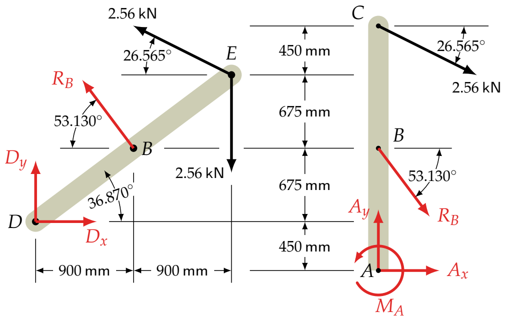
$$
\begin{aligned}
D_x &= 1.9997\,\text{kN} \\
D_y &= 1.8019\,\text{kN} \\
R_D &= 2.6918\,\text{kN} \\
&\quad \text{at } 42.022^\circ \\
R_B &= -0.48347\,\text{kN}
\end{aligned}
$$

Example 4: The Solution

4. Sum the moments about D to find R_B.
5. Sum the x- and y-components of the forces acting on DBE to find R_D.
6. The components of R_D are positive so the reaction is in the first quadrant.

$$
\begin{aligned}
R_D &= \sqrt{(1.9997\,\text{kN})^2 + (1.8019\,\text{kN})^2} \\
&= 2.6918\,\text{kN}
\end{aligned}
$$

$$
R_D\theta = \tan^{-1}\left[\frac{1.8019}{1.9997}\right] = 42.022^\circ
$$



$$M_A = 4.8256\,\text{kN}\cdot\text{m}$$

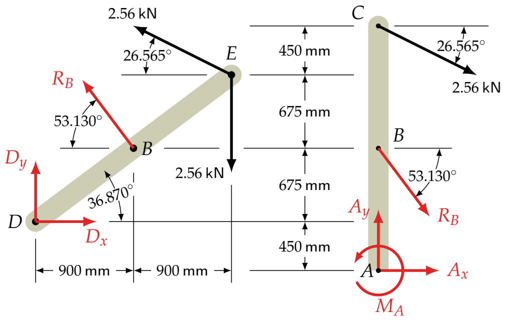
$$D_x = 1.9997\,\text{kN}$$
$$D_y = 1.8019\,\text{kN}$$
$$R_D = 2.6918\,\text{kN} \text{ at } 42.022^\circ$$

$$R_B = -0.48347\,\text{kN}$$

Example 4: The Solution

4. Sum the moments about D to find R_B.
5. Sum the x- and y-components of the forces acting on DBE to find R_D.
6. The components of R_D are positive so the reaction is in the first quadrant.
7. Member ABC: Take moments about A to find the reacting moment at the fixed connection.

$$\begin{aligned}\Sigma M_A &= -(-0.48347\,\text{kN})\cdot\cos 53.130^\circ \times 1125\,\text{mm} \\ &\quad - 2.56\,\text{kN}\cdot\cos 26.565^\circ \times 2250\,\text{mm} \\ &\quad + M_A = 0 \\ \Rightarrow M_A &= 4825.6\,\text{kN}\cdot\text{mm} = 4.8256\,\text{kN}\cdot\text{m}\end{aligned}$$



$$
\begin{aligned}
A_x &= -1.9997\,\text{kN} \\
A_y &= 0.75809\,\text{kN} \\
M_A &= 4.8256\,\text{kN·m}
\end{aligned}
$$

$$
\begin{aligned}
D_x &= 1.9997\,\text{kN} \\
D_y &= 1.8019\,\text{kN} \\
R_D &= 2.6918\,\text{kN} \\
&\quad \text{at } 42.022^\circ
\end{aligned}
$$

$$R_B = -0.48347\,\text{kN}$$

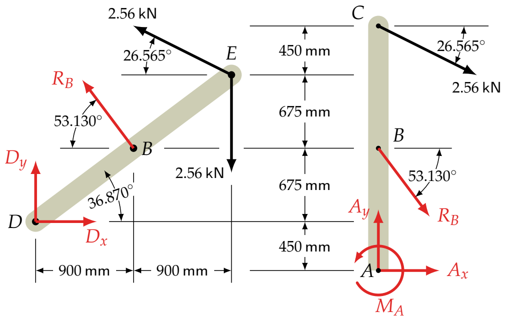
Example 4: The Solution

4. Sum the moments about D to find R_B.
5. Sum the x- and y-components of the forces acting on DBE to find R_D.
6. The components of R_D are positive so the reaction is in the first quadrant.
7. Member ABC: Take moments about A to find the reacting moment at the fixed connection.
8. Sum the components acting on ABC to find the reaction at A.

$$
\begin{aligned}
\Sigma M_A &= -(-0.48347\,\text{kN})\cdot\cos 53.130^\circ \times 1125\,\text{mm} \\
&\quad - 2.56\,\text{kN}\cdot\cos 26.565^\circ \times 2250\,\text{mm} \\
&\quad + M_A = 0 \\
\Rightarrow M_A &= 4825.6\,\text{kN·mm} = 4.8256\,\text{kN·m}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= A_x + (-0.48347\,\text{kN})\cdot\cos 53.130^\circ \\
&\quad + 2.56\,\text{kN}\cdot\cos 26.565^\circ = 0 \\
\Rightarrow A_x &= -1.9997\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_y &= A_y - (-0.48347\,\text{kN})\cdot\sin 53.130^\circ \\
&\quad - 2.56\,\text{kN}\cdot\sin 26.565^\circ = 0 \\
\Rightarrow A_y &= 0.75809\,\text{kN}
\end{aligned}
$$



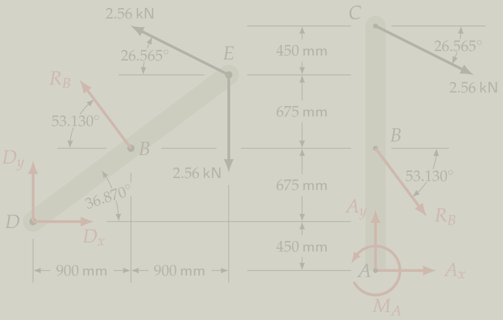
$$
\begin{aligned}
A_x &= -1.9997\,\text{kN}\\
A_y &= 0.75809\,\text{kN}\\
M_A &= 4.8256\,\text{kN}\cdot\text{m}\\
R_A &= 2.1386,\text{kN}\\
&\quad \text{at } 159.24^\circ\\[6pt]
D_x &= 1.9997\,\text{kN}\\
D_y &= 1.8019\,\text{kN}\\
R_D &= 2.6918\,\text{kN}\\
&\quad \text{at } 42.022^\circ\\[6pt]
R_B &= -0.48347\,\text{kN}
\end{aligned}
$$

Example 4: The Solution

4. Sum the moments about D to find R_B.
5. Sum the x- and y-components of the forces acting on DBE to find R_D.
6. The components of R_D are positive so the reaction is in the first quadrant.
7. Member ABC: Take moments about A to find the reacting moment at the fixed connection.
8. Sum the components acting on ABC to find the reaction at A.

$$
\begin{aligned}
R_A &= \sqrt{(-1.9997\,\text{kN})^2 + (0.75809\,\text{kN})^2}\\
&= 2.1386\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
R_A\theta &= 180^\circ - \tan^{-1}\left[\frac{0.75809}{1.9997}\right]\\
&= 159.24^\circ
\end{aligned}
$$



$A_x = -1.9997$ kN
$A_y = 0.75809$ kN
$M_A = 4.8256$ kN·m
$R_A = 2.1386$, kN at 159.24°

$D_x = 1.9997$ kN
$D_y = 1.8019$ kN
$R_D = 2.6918$ kN at 42.022°

$R_B = -0.48347$ kN

Example 4: The Solution

4. Sum the moment
5. Sum the x- and acting on DBE to
6. The components reaction is in the
7. Member ABC: Take moments about A to find the reacting moment at the fixed connection.
8. Sum the components acting on ABC to find the reaction at A.

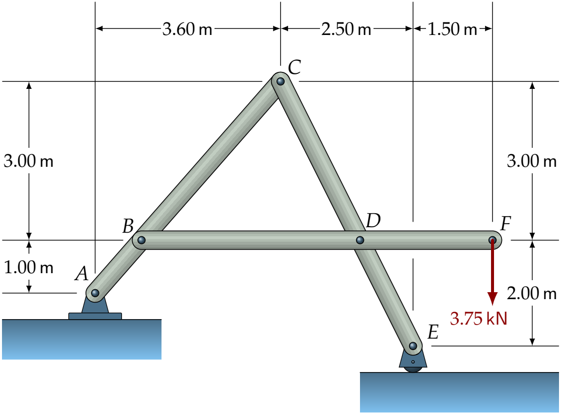
$R_A = \sqrt{(-1.9997\,\text{kN})^2 + (0.75809\,\text{kN})^2}$

159.24°

The Answers

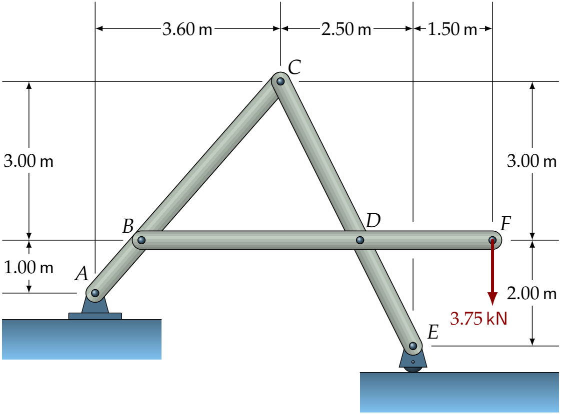
The reaction at A is 2.14 kN at 159°, with a reacting moment of 4.83 kN·m.

The reaction at D is 2.69 kN at 42.0°.



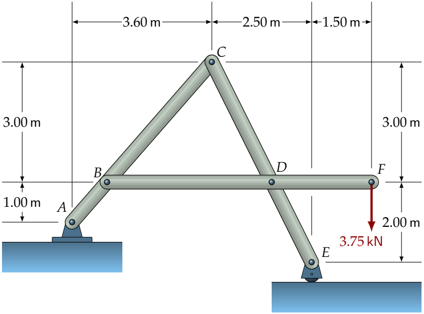
Complex Frames: Example 5

Determine the magnitude of the reactions at A, B, C, D and E.



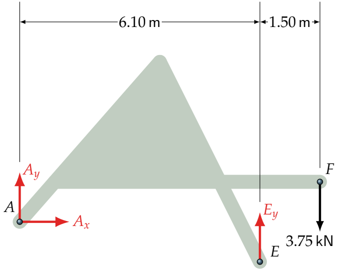
Example 5: Our Method

- Each of the four pinned connections A, B, C and D has two components. E has only one. There are a total of 9 unknowns, and three members.
- This example has three support unknowns. We can find those first, before breaking the frame apart and investigating individual members.



Example 5: The Solution

1. Find the support reactions first.



$$A_x = 0$$
$$A_y = -0.92213\,\text{kN}$$

$$E_y = 4.6721\,\text{kN}$$

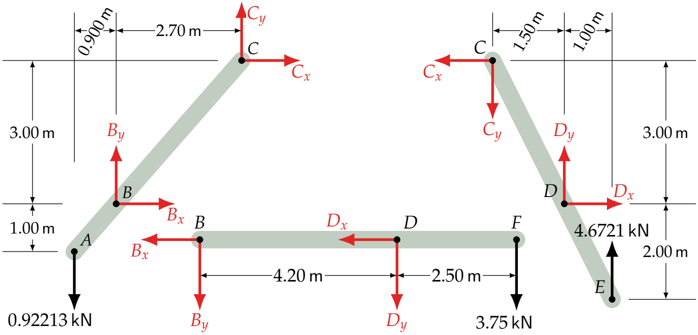
Example 5: The Solution

1. Find the support reactions first.

$$\Sigma M_A = E_y \cdot 6.10\,\text{m} - 3.75\,\text{kN} \cdot 7.60\,\text{m} = 0$$
$$\Rightarrow E_y = 4.6721\,\text{kN}$$

$$\Sigma F_x = A_x = 0$$

$$\Sigma F_y = A_y + 4.6721\,\text{kN} - 3.75\,\text{kN} = 0$$
$$\Rightarrow A_y = -0.9221\,\text{kN}$$



$A_x = 0$

$A_y = -0.92213\,\text{kN}$

$E_y = 4.6721\,\text{kN}$

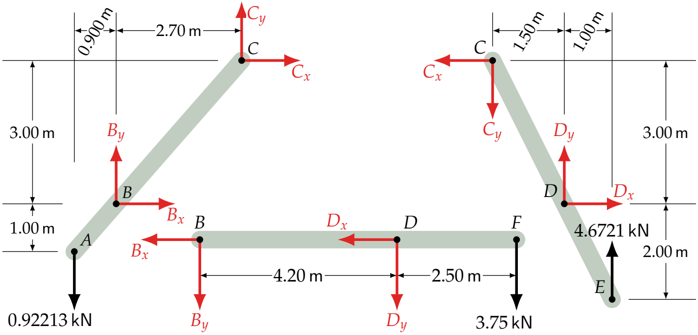
Example 5: The Solution

1. Find the support reactions first.
2. Separate the members and draw the FBDs, using similar triangles to find the various missing horizontal distances.

$$\Sigma M_A = E_y \cdot 6.10\,\text{m} - 3.75\,\text{kN} \cdot 7.60\,\text{m} = 0$$
$$\Rightarrow E_y = 4.6721\,\text{kN}$$

$$\Sigma F_x = A_x = 0$$

$$\Sigma F_y = A_y + 4.6721\,\text{kN} - 3.75\,\text{kN} = 0$$
$$\Rightarrow A_y = -0.9221\,\text{kN}$$



$A_x = 0$
$A_y = -0.92213\,\text{kN}$

$B_y = 2.2321\,\text{kN}$

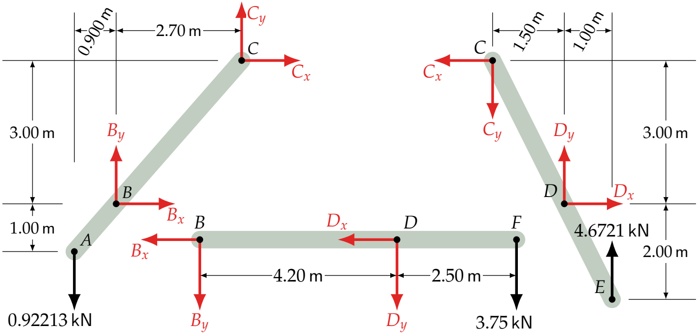
$D_y = -5.9821\,\text{kN}$
$E_y = 4.6721\,\text{kN}$

Example 5: The Solution

1. Find the support reactions first.
2. Separate the members and draw the FBDs, using similar triangles to find the various missing horizontal distances.
3. None of the three members is directly solvable but we can find B_y and D_y from member BDF.

$$\Sigma M_B = -D_y \times 4.20\,\text{m} - 3.75\,\text{kN} \times 6.70\,\text{m} = 0$$
$$\Rightarrow D_y = -5.9821\,\text{kN}$$

$$\Sigma F_y = -B_y - (-5.9821\,\text{kN}) - 3.75\,\text{kN} = 0$$
$$\Rightarrow B_y = 2.2321\,\text{kN}$$



$$A_x = 0$$
$$A_y = -0.92213\,\text{kN}$$
$$B_y = 2.2321\,\text{kN}$$
$$D_x = -0.90237\,\text{kN}$$
$$D_y = -5.9821\,\text{kN}$$
$$E_y = 4.6721\,\text{kN}$$

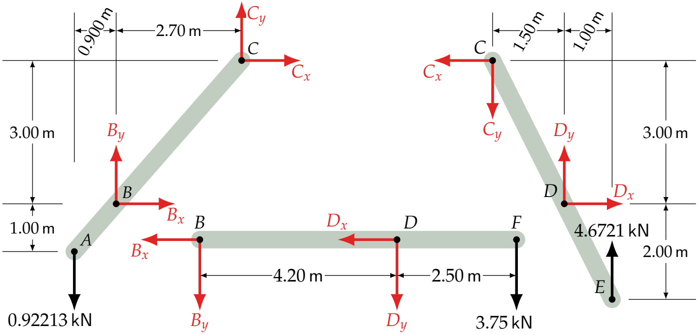
Example 5: The Solution

1. Find the support reactions first.
2. Separate the members and draw the FBDs, using similar triangles to find the various missing horizontal distances.
3. None of the three members is directly solvable but we can find B_y and D_y from member BDF.
4. Now, considering member CDE, take moments about C to calculate D_x.

$$\Sigma M_B = -D_y \times 4.20\,\text{m} - 3.75\,\text{kN} \times 6.70\,\text{m} = 0$$
$$\Rightarrow D_y = -5.9821\,\text{kN}$$

$$\Sigma F_y = -B_y - (-5.9821\,\text{kN}) - 3.75\,\text{kN} = 0$$
$$\Rightarrow B_y = 2.2321\,\text{kN}$$

$$\begin{aligned}\Sigma M_C &= (-5.9821\,\text{kN}) \times 1.50\,\text{m}\\ &\quad + D_x \times 3.00\,\text{m}\\ &\quad + 4.6721\,\text{kN} \times 2.50\,\text{m} = 0\end{aligned}$$
$$\Rightarrow D_x = -0.90237\,\text{kN}$$



$$A_x = 0$$
$$A_y = -0.92213\,\text{kN}$$

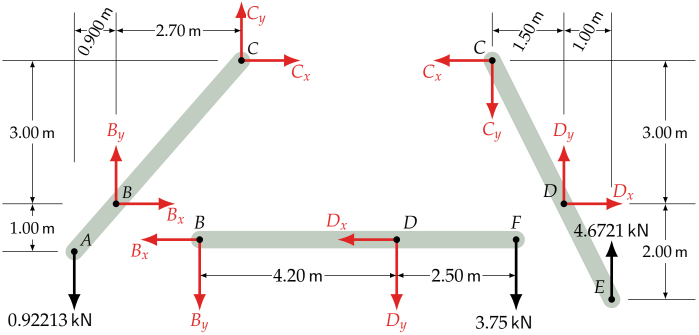
$$B_y = 2.2321\,\text{kN}$$
$$C_x = -0.90237\,\text{kN}$$
$$C_y = -1.3100\,\text{kN}$$
$$D_x = -0.90237\,\text{kN}$$
$$D_y = -5.9821\,\text{kN}$$
$$E_y = 4.6721\,\text{kN}$$

Example 5: The Solution

5. Now, summing components on member *CDE* will give the reaction components at *C*.

$$\Sigma F_x = D_x - C_x = 0$$
$$\Rightarrow C_x = D_x = -0.90237\,\text{kN}$$

$$\Sigma F_y = -C_y + D_y + 4.6721\,\text{kN} = 0$$
$$\Rightarrow C_y = -5.9821\,\text{kN} + 4.6721\,\text{kN}$$
$$= -1.3100\,\text{kN}$$



$$
\begin{aligned}
A_x &= 0 \\
A_y &= -0.92213\,\text{kN} \\
B_x &= 0.90237\,\text{kN} \\
B_y &= 2.2321\,\text{kN} \\
C_x &= -0.90237\,\text{kN} \\
C_y &= -1.3100\,\text{kN} \\
D_x &= -0.90237\,\text{kN} \\
D_y &= -5.9821\,\text{kN} \\
E_y &= 4.6721\,\text{kN}
\end{aligned}
$$

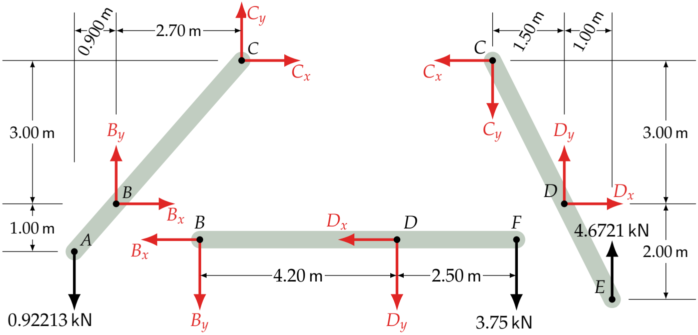
Example 5: The Solution

5. Now, summing components on member *CDE* will give the reaction components at *C*.
6. If we return to member *BDF*, we can find the last unknown component: B_x.

$$
\begin{aligned}
\Sigma F_x &= D_x - C_x = 0 \\
\Rightarrow C_x &= D_x = -0.90237\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_y &= -C_y + D_y + 4.6721\,\text{kN} = 0 \\
\Rightarrow C_y &= -5.9821\,\text{kN} + 4.6721\,\text{kN} \\
&= -1.3100\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= -B_x - D_x = 0 \\
\Rightarrow B_x &= -D_x = 0.90237\,\text{kN}
\end{aligned}
$$



$A_x = 0$
$A_y = -0.92213\,\text{kN}$
$B_x = 0.90237\,\text{kN}$
$B_y = 2.2321\,\text{kN}$
$C_x = -0.90237\,\text{kN}$
$C_y = -1.3100\,\text{kN}$
$D_x = -0.90237\,\text{kN}$
$D_y = -5.9821\,\text{kN}$
$E_y = 4.6721\,\text{kN}$

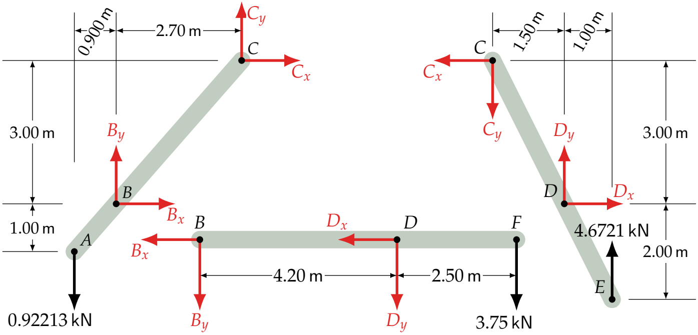
Example 5: The Solution

5. Now, summing components on member *CDE* will give the reaction components at *C*.
6. If we return to member *BDF*, we can find the last unknown component: B_x.
7. After so many calculations, it is good practice to do a check. Considering member *ABC*, moments about *A* should sum to zero (apart from some minor rounding errors).

$$
\begin{aligned}
\Sigma F_x &= D_x - C_x = 0 \\
\Rightarrow C_x &= D_x = -0.90237\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_y &= -C_y + D_y + 4.6721\,\text{kN} = 0 \\
\Rightarrow C_y &= -5.9821\,\text{kN} + 4.6721\,\text{kN} \\
&= -1.3100\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= -B_x - D_x = 0 \\
\Rightarrow B_x &= -D_x = 0.90237\,\text{kN}
\end{aligned}
$$

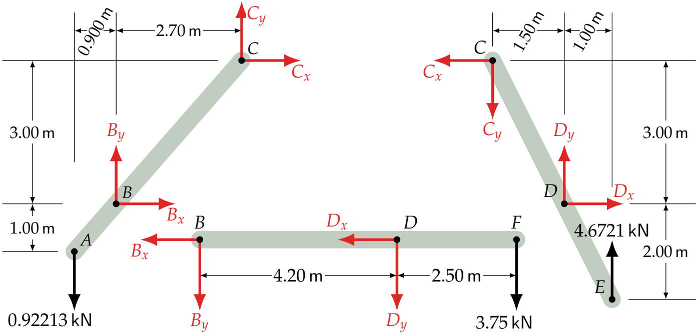


$$
\begin{aligned}
A_x &= 0 \\
A_y &= -0.92213\,\text{kN} \\
B_x &= 0.90237\,\text{kN} \\
B_y &= 2.2321\,\text{kN} \\
C_x &= -0.90237\,\text{kN} \\
C_y &= -1.3100\,\text{kN} \\
D_x &= -0.90237\,\text{kN} \\
D_y &= -5.9821\,\text{kN} \\
E_y &= 4.6721\,\text{kN}
\end{aligned}
$$

Example 5: The Solution

8. We have all the components.
   Now, find the reaction magnitudes.

$$
\begin{aligned}
R_A &= 0.92213\,\text{kN} \\
R_B &= \sqrt{(0.90237\,\text{kN})^2 + (2.2321\,\text{kN})^2} \\
&= 2.4076\,\text{kN} \\
R_C &= \sqrt{(-0.90237\,\text{kN})^2 + (-1.3100\,\text{kN})^2} \\
&= 1.5907\,\text{kN} \\
R_D &= \sqrt{(-0.90237\,\text{kN})^2 + (-5.9821\,\text{kN})^2} \\
&= 6.0498\,\text{kN} \\
R_E &= 4.6721\,\text{kN}
\end{aligned}
$$

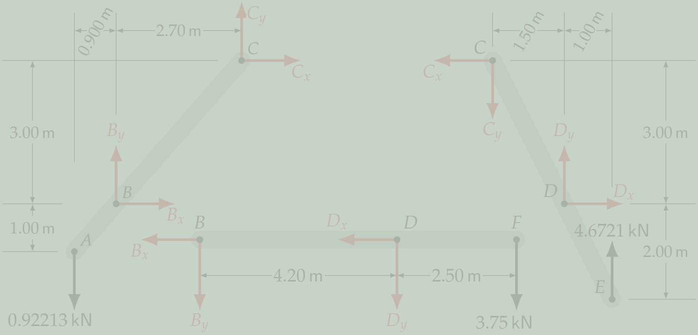


$$
\begin{aligned}
A_x &= 0 \\
A_y &= -0.92213\,\text{kN} \\
B_x &= 0.90237\,\text{kN} \\
B_y &= 2.2321\,\text{kN} \\
C_x &= -0.90237\,\text{kN} \\
C_y &= -1.3100\,\text{kN} \\
D_x &= -0.90237\,\text{kN} \\
D_y &= -5.9821\,\text{kN} \\
E_y &= 4.6721\,\text{kN}
\end{aligned}
$$

Example 5: The Solution

8. We have all the components.
   Now, find the reaction magnitudes.

$$
\begin{aligned}
R_A &= 0.92213\,\text{kN} \\
R_B &= \sqrt{(0.90237\,\text{kN})^2 + (2.2321\,\text{kN})^2} \\
&= 2.4076\,\text{kN} \\
R_C &= \sqrt{(-0.90237\,\text{kN})^2 + (-1.3100\,\text{kN})^2} \\
&= 1.5907\,\text{kN} \\
R_D &= \sqrt{(-0.90237\,\text{kN})^2 + (-5.9821\,\text{kN})^2} \\
&= 6.0498\,\text{kN} \\
R_E &= 4.6721\,\text{kN}
\end{aligned}
$$



$A_x = 0$

$A_y = -0.92213\,\text{kN}$

$B_x = 0.90237\,\text{kN}$

$B_y = 2.2321\,\text{kN}$

$C_x = -0.90237\,\text{kN}$

$C_y = -1.3100\,\text{kN}$

$D_x = -0.90237\,\text{kN}$

$D_y = -5.9821\,\text{kN}$

$E_y = 4.6721\,\text{kN}$

Example 5: The Solution

8. We have all the
Now, find the re

The Answers

$$R_A = 0.922\,\text{kN}$$
$$R_B = 2.41\,\text{kN}$$
$$R_C = 1.59\,\text{kN}$$
$$R_D = 6.05\,\text{kN}$$
$$R_E = 4.67\,\text{kN}$$

213 kN

$.90237\,\text{kN})^2 + (2.2321\,\text{kN})^2$

76 kN

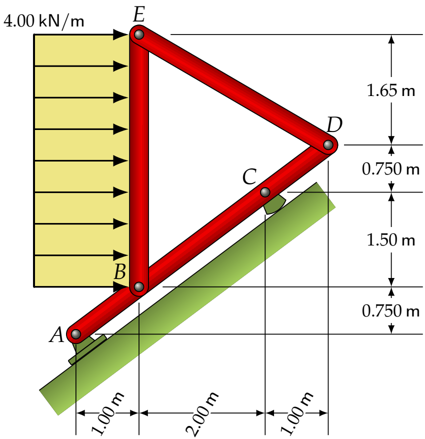
$-0.90237\,\text{kN})^2 + (-1.3100\,\text{kN})^2$

07 kN

$-0.90237\,\text{kN})^2 + (-5.9821\,\text{kN})^2$

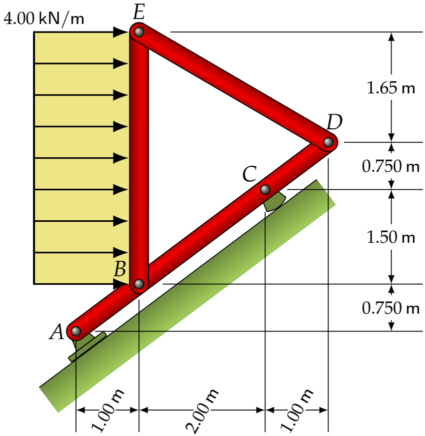
98 kN

21 kN



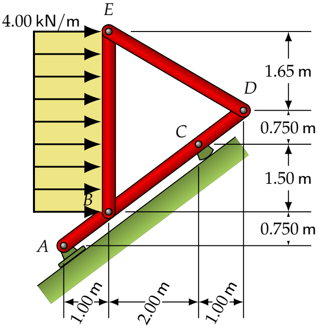
Complex Frames: Example 6

Determine the magnitude of the reactions at A and C, and the force in two-force member DE.



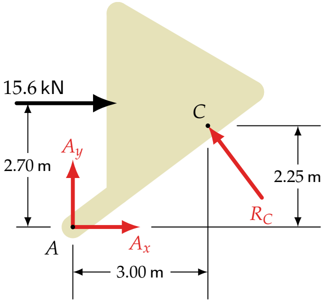
Example 6: Our Method

- Each of the four pinned connections A, B, D and E has two unknown components. C has only one.
  There are a total of 9 unknowns for the three members.
- We can find the three support unknowns first, before breaking the frame apart and investigating individual members.



Example 6: The Solution

1. Find the support reactions first.



$R_C = 11.232\,\text{kN at } 225.40^\circ$

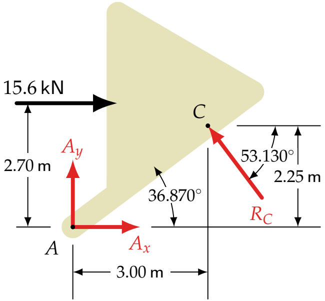
Example 6: The Solution

1. Find the support reactions first.
   1.1 The reaction at C

$$|AC| = \sqrt{(3.00\,\text{m})^2 + (2.25\,\text{m})^2} = 3.75\,\text{m}$$

$$\Sigma M_A = R_C \times 3.75\,\text{m} - 15.6\,\text{kN} \times 2.70\,\text{m} = 0$$
$$\Rightarrow R_C = 11.232\,\text{kN}$$

$$AC_\theta = \tan^{-1}\left[\frac{2.25}{3.00}\right] = 36.870^\circ$$
$$90^\circ - 36.870^\circ = 53.130^\circ$$
$$R_{C\theta} = 180^\circ - 53.130^\circ = 126.87^\circ$$



$$A_x = -8.8608\,\text{kN}$$
$$A_y = -8.9856\,\text{kN}$$
$$R_A = 12.620\,\text{kN at } 134.24^\circ$$

$$R_C = 11.232\,\text{kN at } 225.40^\circ$$

Example 6: The Solution

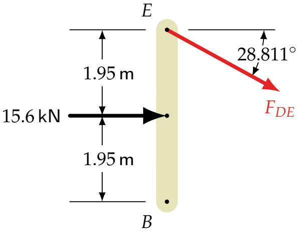
1. Find the support reactions first.
   1.1 The reaction at C
   1.2 The reaction at A

$$\Sigma F_x = A_x + R_C \cdot \cos 126.87^\circ = 0$$
$$\Rightarrow A_x = -8.8608\,\text{kN}$$

$$\Sigma F_y = A_y + R_C \cdot \sin 126.87^\circ - 15.6\,\text{kN} = 0$$
$$\Rightarrow A_y = -8.9856\,\text{kN}$$

$$R_A = \sqrt{(-8.8608\,\text{kN})^2 + (-8.9856\,\text{kN})^2}$$
$$= 12.620\,\text{kN}$$

$$R_{A\theta} = 180^\circ + \tan^{-1}\left[\frac{8.9856}{8.8608}\right] = 225.40^\circ$$



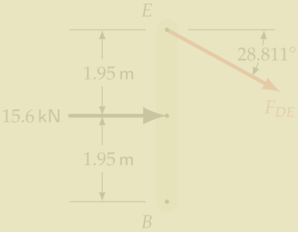
$$A_x = -8.8608\,\text{kN}$$
$$A_y = -8.9856\,\text{kN}$$
$$R_A = 12.620\,\text{kN at } 134.24^\circ$$
$$R_C = 11.232\,\text{kN at } 225.40^\circ$$
$$F_{DE} = 8.9019\,\text{kN in compression}$$

Example 6: The Solution

1. Find the support reactions first.
   1.1 The reaction at C
   1.2 The reaction at A
2. To investigate member DE, we need only consider member BE

$$DE_\theta = \tan^{-1}\left[\frac{1.65\,\text{m}}{3.00\,\text{m}}\right] = 28.811^\circ$$

$$\begin{aligned}\Sigma M_B &= -F_{DE}\cdot\cos 28.811^\circ \times 3.90\,\text{m}\\ &\quad - 15.6\,\text{kN}\times 1.95\,\text{m} = 0\\ \Rightarrow F_{DE} &= -8.9019\,\text{kN}\end{aligned}$$



$A_x = -8.8608\,\text{kN}$

$A_y = -8.9856\,\text{kN}$

$R_A = 12.620\,\text{kN at } 134.24°$

$R_C = 11.232\,\text{kN at } 225.40°$

$F_{DE} = 8.9019\,\text{kN in compression}$

Example 6: The Solution

1. Find the support reactions first
   1.1 The reaction at C
   1.2 The reaction at A
2. To investigate member DE, we
   only consider member BE

The Answers

$$R_A = 12.6\,\text{kN at } 134°$$

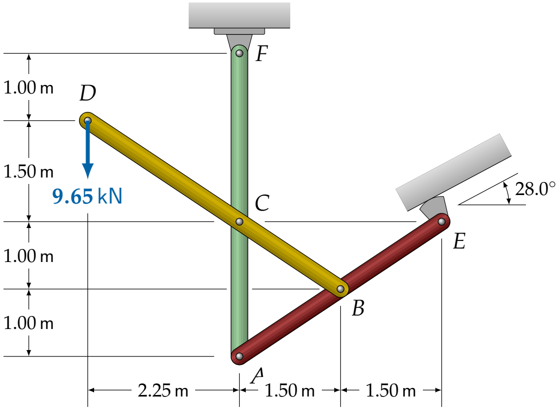
$$R_C = 11.2\,\text{kN at } 225°$$

$$F_{DE} = 8.90\,\text{kN (Compression)}$$

$= 28.811°$

$11° \times 3.90\,\text{m}$

$.95\,\text{m} = 0$

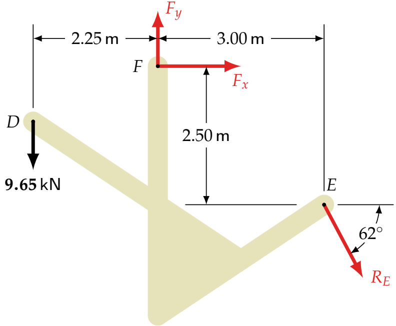


Complex Frames: Example 7

Determine the magnitude of reactions in A, B and C due to the 9.65 kN load at D.

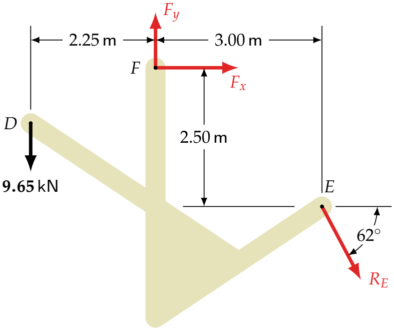
Example 7: The Solution

1. Find the support reactions first.



Example 7: The Solution

1. Find the support reactions first.
   1.1 The roller reaction at E



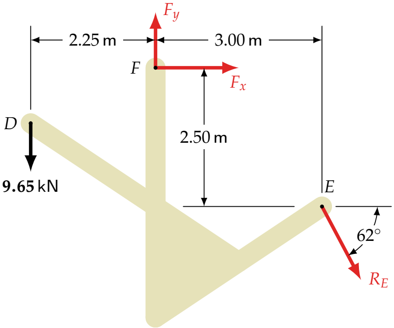
$$\Sigma M_F = 9.65\,\text{kN} \times 2.25\,\text{m} - R_E \cdot \sin 62^\circ \times 3.00\,\text{m} + R_E \cdot \cos 62^\circ \times 2.50\,\text{m} = 0$$

$$\Rightarrow R_E = \frac{9.65\,\text{kN} \times 2.25\,\text{m}}{\sin 62^\circ \times 3.00\,\text{m} - \cos 62^\circ \times 2.50\,\text{m}} = 14.719\,\text{kN}$$

$$R_E = 14.719\,\text{kN at } 298^\circ$$

Example 7: The Solution

1. Find the support reactions first.
   1.1 The roller reaction at E
   1.2 The reaction at F



$$\Sigma M_F = 9.65\,\text{kN} \times 2.25\,\text{m} - R_E \cdot \sin 62^\circ \times 3.00\,\text{m} + R_E \cdot \cos 62^\circ \times 2.50\,\text{m} = 0$$

$$\Rightarrow R_E = \frac{9.65\,\text{kN} \times 2.25\,\text{m}}{\sin 62^\circ \times 3.00\,\text{m} - \cos 62^\circ \times 2.50\,\text{m}} = 14.719\,\text{kN}$$

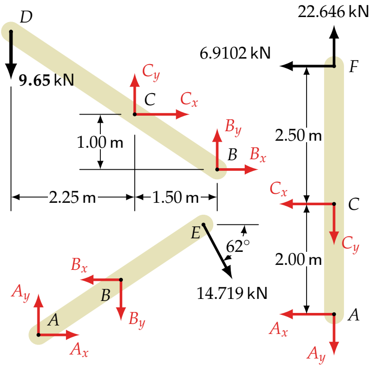
$$\Sigma F_x = F_x + 14.719\,\text{kN} \cdot \cos 62^\circ = 0$$
$$\Rightarrow F_x = -6.9102\,\text{kN}$$

$$\Sigma F_y = F_y - 9.65\,\text{kN} - 14.719\,\text{kN} \cdot \sin 62^\circ = 0$$
$$\Rightarrow F_y = 22.646\,\text{kN}$$

$$R_E = 14.719\,\text{kN at } 298^\circ$$
$$F_x = -6.9102\,\text{kN}$$
$$F_y = 22.646\,\text{kN}$$

Example 7: The Solution

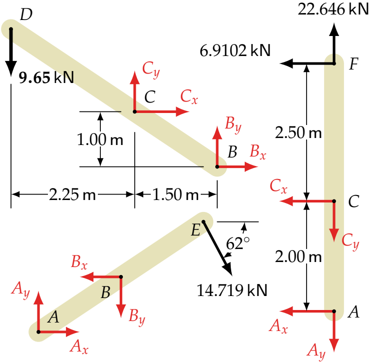
1. Find the support reactions first.
   1.1 The roller reaction at E
   1.2 The reaction at F
2. Draw FBDs for the individual frame members.



$$R_E = 14.719\,\text{kN at } 298^\circ$$
$$F_x = -6.9102\,\text{kN}$$
$$F_y = 22.646\,\text{kN}$$

Example 7: The Solution

1. Find the support reactions first.
   1.1 The roller reaction at E
   1.2 The reaction at F
2. Draw FBDs for the individual frame members.
3. None of the members is directly solvable but, considering ACF, moments about A will give C_x. Switching to member BCD, we can find BCD's three remaining components.



From member ACF:

$$\Sigma M_A = C_x \times 2.00\,\text{m} + 6.9102\,\text{kN} \times 4.50\,\text{m} = 0$$
$$\Rightarrow C_x = -15.548\,\text{kN}$$

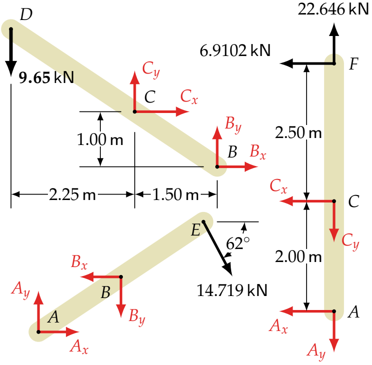
From member BCD:

$$\Sigma M_B = 15.548\,\text{kN} \times 1.00\,\text{m} - C_y \times 1.50\,\text{m} + 9.65\,\text{kN} \times 3.75\,\text{m} = 0$$
$$\Rightarrow C_y = 34.490\,\text{kN}$$

$$C_x = -15.548\,\text{kN}$$
$$C_y = 34.490\,\text{kN}$$
$$R_E = 14.719\,\text{kN at } 298^\circ$$
$$F_x = -6.9102\,\text{kN}$$
$$F_y = 22.646\,\text{kN}$$

Example 7: The Solution

1. Find the support reactions first.
   1.1 The roller reaction at E
   1.2 The reaction at F
2. Draw FBDs for the individual frame members.
3. None of the members is directly solvable but, considering ACF, moments about A will give C_x. Switching to member BCD, we can find BCD's three remaining components.



From member ACF:

$$\Sigma M_A = C_x \times 2.00\,\text{m} + 6.9102\,\text{kN} \times 4.50\,\text{m} = 0$$
$$\Rightarrow C_x = -15.548\,\text{kN}$$

From member BCD:

$$\Sigma M_B = 15.548\,\text{kN} \times 1.00\,\text{m} - C_y \times 1.50\,\text{m} + 9.65\,\text{kN} \times 3.75\,\text{m} = 0$$
$$\Rightarrow C_y = 34.490\,\text{kN}$$

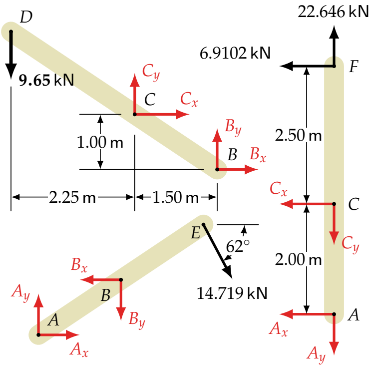
$$\Sigma F_x = B_x - 15.548\,\text{kN} = 0$$
$$\Rightarrow B_x = 15.548\,\text{kN}$$

$$\Sigma F_y = B_y + 34.490\,\text{kN} - 9.65\,\text{kN} = 0$$
$$\Rightarrow B_y = -24.840\,\text{kN}$$

$$B_x = 15.548\,\text{kN}$$
$$B_y = -24.840\,\text{kN}$$
$$C_x = -15.548\,\text{kN}$$
$$C_y = 34.490\,\text{kN}$$
$$R_E = 14.719\,\text{kN at } 298^\circ$$
$$F_x = -6.9102\,\text{kN}$$
$$F_y = 22.646\,\text{kN}$$

Example 7: The Solution

1. Find the support reactions first.
   1.1 The roller reaction at E
   1.2 The reaction at F
2. Draw FBDs for the individual frame members.
3. None of the members is directly solvable but, considering ACF, moments about A will give C_x. Switching to member BCD, we can find BCD's three remaining components.



Now, from member ACF (or we could use ABE):

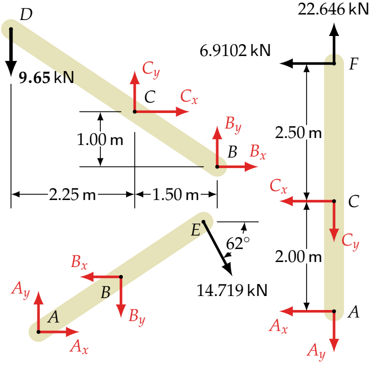
$$\Sigma F_x = -Ax + 15.548\,\text{kN} - 6.9102\,\text{kN} = 0$$
$$\Rightarrow A_x = 8.6378\,\text{kN}$$

$$\Sigma F_y = -A_y - 34.490\,\text{kN} + 22.646\,\text{kN} = 0$$
$$\Rightarrow A_y = -11.844\,\text{kN}$$

$$A_x = -8.6378\,\text{kN}$$
$$A_y = -11.844\,\text{kN}$$
$$B_x = 15.548\,\text{kN}$$
$$B_y = -24.840\,\text{kN}$$
$$C_x = -15.548\,\text{kN}$$
$$C_y = 34.490\,\text{kN}$$
$$R_E = 14.719\,\text{kN at } 298^\circ$$
$$F_x = -6.9102\,\text{kN}$$
$$F_y = 22.646\,\text{kN}$$

Example 7: The Solution

1. Find the support reactions first.
   1.1 The roller reaction at E
   1.2 The reaction at F
2. Draw FBDs for the individual frame members.
3. None of the members is directly solvable but, considering ACF, moments about A will give C_x. Switching to member BCD, we can find BCD's three remaining components.



Now, from member ACF (or we could use ABE):

$$\Sigma F_x = -Ax + 15.548\,\text{kN} - 6.9102\,\text{kN} = 0$$
$$\Rightarrow A_x = 8.6378\,\text{kN}$$

$$\Sigma F_y = -A_y - 34.490\,\text{kN} + 22.646\,\text{kN} = 0$$
$$\Rightarrow A_y = -11.844\,\text{kN}$$

Get the reaction magnitudes:

$$R_A = \sqrt{(-8.6378\,\text{kN})^2 + (-11.844\,\text{kN})^2} = 14.659\,\text{kN}$$

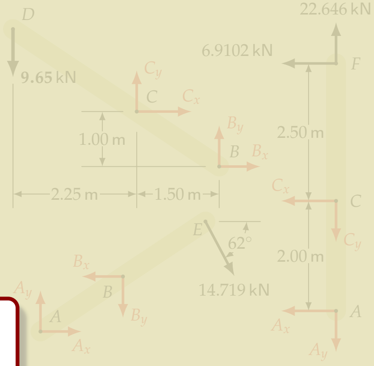
$$R_B = \sqrt{(15.548\,\text{kN})^2 + (-24.840\,\text{kN})^2} = 29.305\,\text{kN}$$

$$R_C = \sqrt{(-15.548\,\text{kN})^2 + (34.490\,\text{kN})^2} = 37.833\,\text{kN}$$

$$A_x = -8.6378\,\text{kN}$$
$$A_y = -11.844\,\text{kN}$$
$$B_x = 15.548\,\text{kN}$$
$$B_y = -24.840\,\text{kN}$$
$$C_x = -15.548\,\text{kN}$$
$$C_y = 34.490\,\text{kN}$$
$$R_E = 14.719\,\text{kN at } 298^\circ$$
$$F_x = -6.9102\,\text{kN}$$
$$F_y = 22.646\,\text{kN}$$

Example 7: The Solution

1. Find the support reactions first.
   1.1 The roller reaction at E
   1.2 The reaction at F
2. Draw FBDs for the individual frame members.
3. None of the members is directly solvable but, considering ACF, moments about A will give C_x. Switching to member BCD, we can find BCD's three remaining components.



Now, from m

$\Sigma F_x = $

$\Rightarrow A_x = 8$

$\Sigma F_y = $

$\Rightarrow A_y = $

Get the reac

$$R_A = \sqrt{(-8.6378\,\text{kN})^2 + (-11.844\,\text{kN})^2} = 14.659\,\text{kN}$$

$$R_B = \sqrt{(15.548\,\text{kN})^2 + (-24.840\,\text{kN})^2} = 29.305\,\text{kN}$$

$$R_C = \sqrt{(-15.548\,\text{kN})^2 + (34.490\,\text{kN})^2} = 37.833\,\text{kN}$$

$A_x = -8.6378\,\text{kN}$
$A_y = -11.844\,\text{kN}$

$B_x = 15.548\,\text{kN}$
$B_y = -24.840\,\text{kN}$

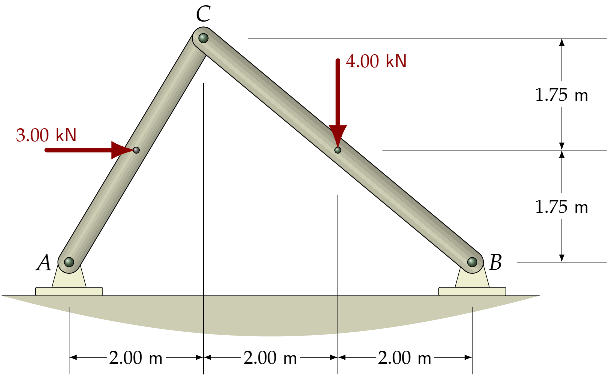
$C_x = -15.548\,\text{kN}$
$C_y = 34.490\,\text{kN}$

$R_E = 14.719\,\text{kN}$ at 298°

$F_x = -6.9102\,\text{kN}$
$F_y = 22.646\,\text{kN}$

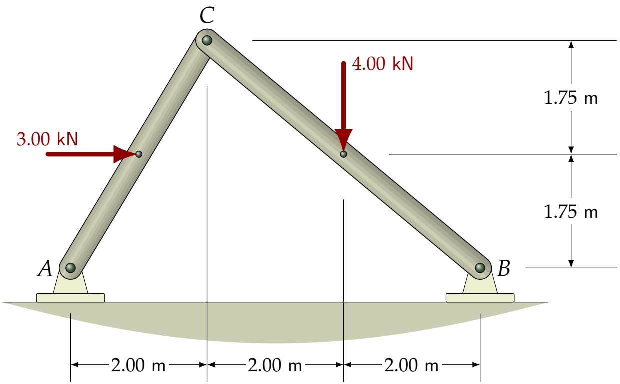
The Answers

$$R_A = 14.7\,\text{kN}$$
$$R_B = 29.3\,\text{kN}$$
$$R_C = 37.8\,\text{kN}$$



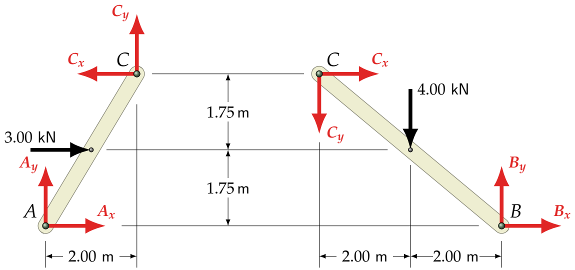
Complex Frames: Example 8

Determine the components of the reactions at A, B and C.



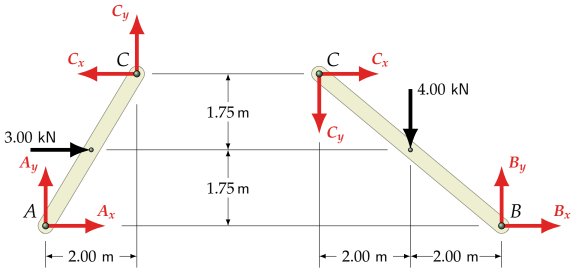
Example 8: Our Method

1. There are too many (four) unknowns to solve the entire frame at once.
2. Draw separate FBDs for each frame member.
3. Summing moments about A and about B will yield two equations in the two unknowns C_x and C_y, which can then be solved.



Example 8: The Solution

1. Draw FBDs of the frame members.

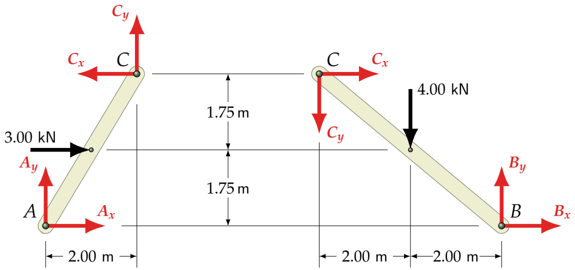


Example 8: The Solution

1. Draw FBDs of the frame members.
2. Sum moments about A and B to get two expressions involving C_x and C_y.

$$\Sigma M_A = C_x \cdot 3.50\,\text{m} + C_y \cdot 2.00\,\text{m} - 3.00\,\text{kN} \times 1.75\,\text{m} = 0$$

$$\Sigma M_B = -C_x \cdot 3.50\,\text{m} + C_y \cdot 4.00\,\text{m} + 4.00\,\text{kN} \times 2.00\,\text{m} = 0$$



Example 8: The Solution

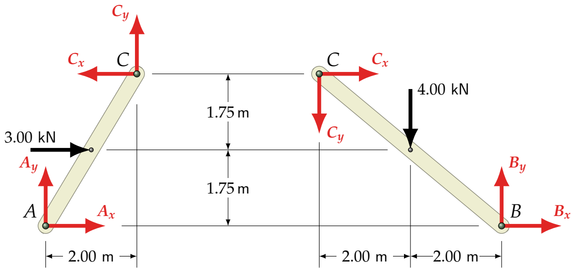
1. Draw FBDs of the frame members.
2. Sum moments about A and B to get two expressions involving C_x and C_y.
3. Rearrange and simplify...

$$\Sigma M_A = C_x \cdot 3.50\,\text{m} + C_y \cdot 2.00\,\text{m} - 3.00\,\text{kN} \times 1.75\,\text{m} = 0$$

$$\Sigma M_B = -C_x \cdot 3.50\,\text{m} + C_y \cdot 4.00\,\text{m} + 4.00\,\text{kN} \times 2.00\,\text{m} = 0$$

$$3.50\,C_x + 2.00\,C_y = 5.2500\,\text{kN}$$

$$3.50\,C_x - 4.00\,C_y = 8.0000\,\text{kN}$$



$C_x = 1.7619\,\text{kN}$
$C_y = -0.45833\,\text{kN}$

Example 8: The Solution

1. Draw FBDs of the frame members.
2. Sum moments about A and B to get two expressions involving C_x and C_y.
3. Rearrange and simplify...
4. Solve for C_x and C_y.

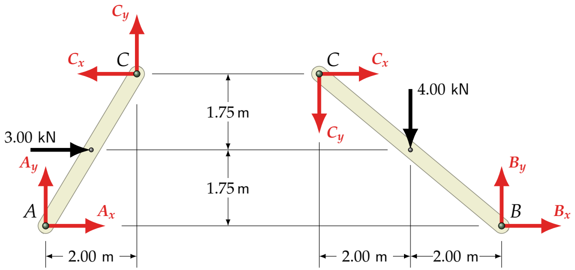
$$\Sigma M_A = C_x \cdot 3.50\,\text{m} + C_y \cdot 2.00\,\text{m} - 3.00\,\text{kN} \times 1.75\,\text{m} = 0$$

$$\Sigma M_B = -C_x \cdot 3.50\,\text{m} + C_y \cdot 4.00\,\text{m} + 4.00\,\text{kN} \times 2.00\,\text{m} = 0$$

$$3.50\,C_x + 2.00\,C_y = 5.2500\,\text{kN}$$

$$3.50\,C_x - 4.00\,C_y = 8.0000\,\text{kN}$$

$$C_x = 1.7619\,\text{kN}, \quad C_y = -0.45833\,\text{kN}$$



$$A_x = -1.2381\,\text{kN}$$
$$A_y = 0.45833\,\text{kN}$$

$$C_x = 1.7619\,\text{kN}$$
$$C_y = -0.45833\,\text{kN}$$

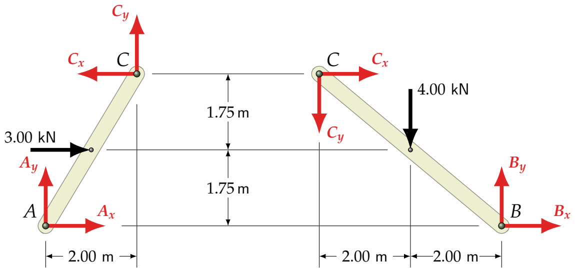
Example 8: The Solution

1. Draw FBDs of the frame members.
2. Sum moments about A and B to get two expressions involving C_x and C_y.
3. Rearrange and simplify...
4. Solve for C_x and C_y.
5. Analyze member AC to find the reaction components at A.

Member AC:

$$\Sigma F_x = A_x + 3.00\,\text{kN} - 1.7619\,\text{kN} = 0$$
$$\Rightarrow A_x = -1.2381\,\text{kN}$$

$$\Sigma F_y = A_y - 0.45833\,\text{kN} = 0$$
$$A_y = 0.45833\,\text{kN} = 0$$



$$A_x = -1.2381\,\text{kN}$$
$$A_y = 0.45833\,\text{kN}$$

$$B_x = -1.7619\,\text{kN}$$
$$B_y = 3.5417\,\text{kN}$$

$$C_x = 1.7619\,\text{kN}$$
$$C_y = -0.45833\,\text{kN}$$

Example 8: The Solution

1. Draw FBDs of the frame members.
2. Sum moments about A and B to get two expressions involving C_x and C_y.
3. Rearrange and simplify...
4. Solve for C_x and C_y.
5. Analyze member AC to find the reaction components at A.
6. Analyze member BC to find the reaction at C

Member AC:

$$\Sigma F_x = A_x + 3.00\,\text{kN} - 1.7619\,\text{kN} = 0$$
$$\Rightarrow A_x = -1.2381\,\text{kN}$$

$$\Sigma F_y = A_y - 0.45833\,\text{kN} = 0$$
$$A_y = 0.45833\,\text{kN} = 0$$

Member BC:

$$\Sigma F_x = B_x + 1.7619\,\text{kN} = 0$$
$$\Rightarrow B_x = -1.7619\,\text{kN}$$

$$\Sigma F_y = B_y - 4.00\,\text{kN} - (-0.45833\,\text{kN}) = 0$$
$$\Rightarrow B_y = 3.5417\,\text{kN}$$

The Answers

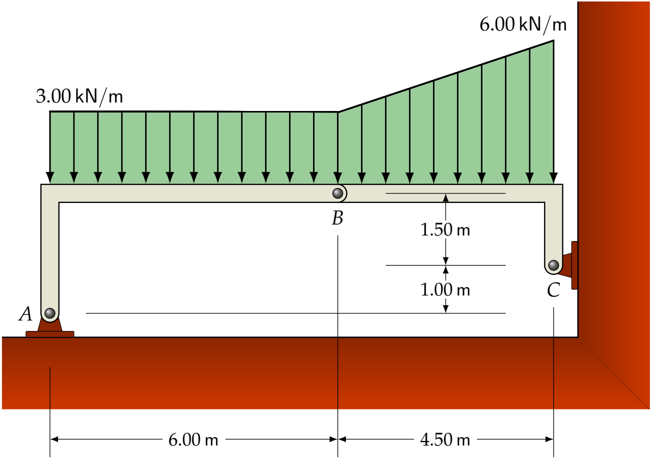
$$A_x = -1.24\,\text{kN}$$
$$A_y = 0.458\,\text{kN}$$

$$B_x = -1.76\,\text{kN}$$
$$B_y = 3.54\,\text{kN}$$

$$C_x = 1.76,\text{kN}$$
$$C_y = -0.458\,\text{kN}$$

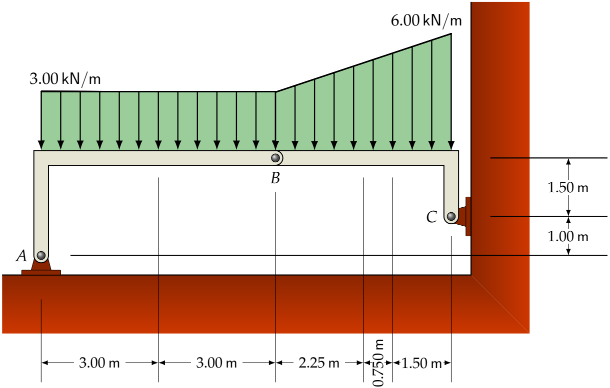
Note that your results may have different signs depending on your assumed directions for the reactions, although if you calculate the magnitudes and angles, they should match for any choice of assumed direction.

Also, specifying the reaction at C may be ambiguous unless you specify the reactive force exerted by which member on the other member. For example, the force exerted on BC by AB, at C, is equal in magnitude but opposite in direction to the force exerted on AC by BC, at C.



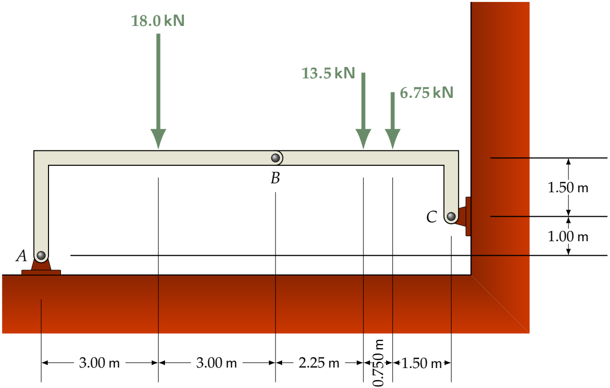
Complex Frames: Example 9

Determine the reactions at A and C.



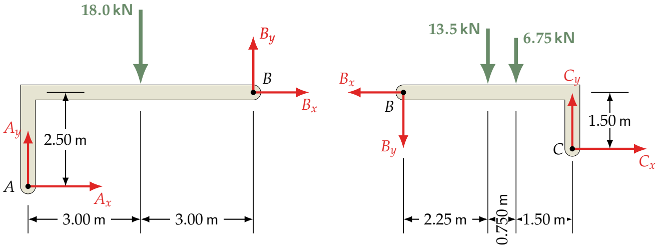
Example 9: The Solution

1. Resolve the distributed loads.



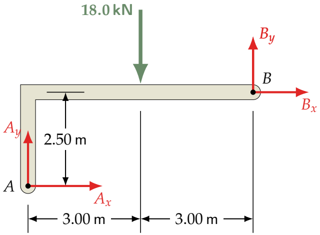
Example 9: The Solution

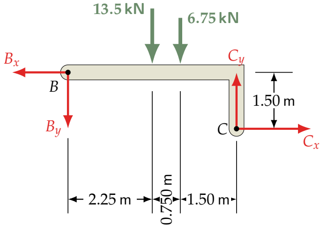
1. Resolve the distributed loads.
2. Draw separated FBDs for the frame members.



Example 9: The Solution

1. Resolve the distributed loads.
2. Draw separated FBDs for the frame members.





$$B_x = -24.000\,\text{kN}$$
$$B_y = -1.0000\,\text{kN}$$

Example 9: The Solution

1. Resolve the distributed loads.
2. Draw separated FBDs for the frame members.
3. Neither member is directly solvable for any unknown. We take moments about A to get an expression containing B_x and B_y, then switch to member BCD to find another expression containing B_x and B_y. The two equations can then be solved simultaneously.

$$\begin{aligned}\Sigma M_A &= B_y \times 6.00\,\text{m} - B_x \times 2.50\,\text{m} \\ &\quad - 18.0\,\text{kN} \times 3.00\,\text{m} = 0\end{aligned}$$

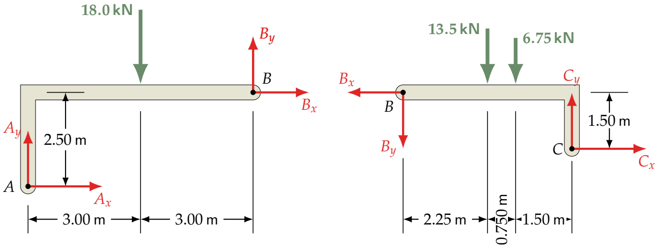
$$\begin{aligned}\Sigma M_C &= B_x \times 1.50\,\text{m} + B_y \times 4.50\,\text{m} \\ &\quad + 13.5\,\text{kN} \times 2.25\,\text{m} \\ &\quad + 6.75\,\text{kN} \times 1.50\,\text{m} = 0\end{aligned}$$

Simplifying and rearranging the two equations:

$$\begin{aligned}-2.50\,B_x + 6.00\,B_y &= 54.0\,\text{kN} \\ 1.50\,B_x + 4.50\,B_y &= -40.5\,\text{kN}\end{aligned}$$

From your calculator:

$$B_x = -24.000\,\text{kN}, \quad B_y = -1.000\,\text{kN}$$



$$R_A = 30.610\,\text{kN} \quad \text{at } 38.367^\circ$$

$$B_x = -24.000\,\text{kN}$$
$$B_y = -1.0000\,\text{kN}$$

Example 9: The Solution

1. Resolve the distributed loads.
2. Draw separated FBDs for the frame members.
3. Neither member is directly solvable for any unknown. We take moments about A to get an expression containing B_x and B_y, then switch to member BCD to find another expression containing B_x and B_y. The two equations can then be solved simultaneously.
4. With B_x and B_y now known, we can sum x- and y-components to get the reactions at A and C.

From frame member AB:

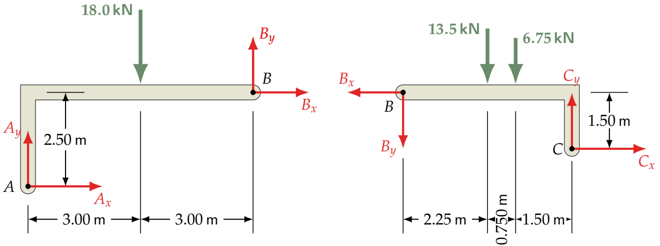
$$\Sigma F_x = A_x - 24.000\,\text{kN} = 0$$
$$\Rightarrow A_x = 24.000\,\text{kN}$$

$$\Sigma F_y = A_y - 18.000\,\text{kN} - 1.0000\,\text{kN} = 0$$
$$\Rightarrow A_y = 19.000\,\text{kN}$$

$$R_A = \sqrt{(24.000\,\text{kN})^2 + (19.000\,\text{kN})^2} = 30.610\,\text{kN}$$

A_x and A_y are both positive, so the reaction is in the first quadrant.

$$R_{A\theta} = \tan^{-1}\left[\frac{19.000}{24.000}\right] = 38.367^\circ$$



$$R_A = 30.610\,\text{kN} \text{ at } 38.367^\circ$$

$$B_x = -24.000\,\text{kN}$$
$$B_y = -1.0000\,\text{kN}$$

$$R_A = 30.766\,\text{kN} \text{ at } 141.27^\circ$$

Example 9: The Solution

1. Resolve the distributed loads.
2. Draw separated FBDs for the frame members.
3. Neither member is directly solvable for any unknown. We take moments about A to get an expression containing B_x and B_y, then switch to member BCD to find another expression containing B_x and B_y. The two equations can then be solved simultaneously.
4. With B_x and B_y now known, we can sum x- and y-components to get the reactions at A and C.

From frame member BC:

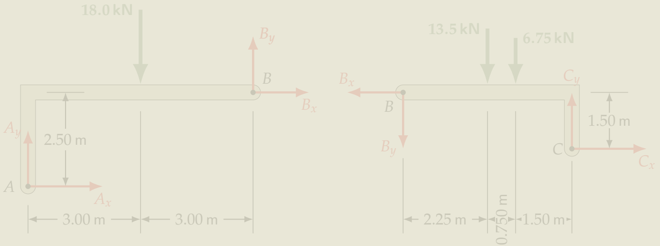
$$\begin{aligned} \Sigma F_x &= C_x + 24.000\,\text{kN} = 0 \\ \Rightarrow C_x &= -24.000\,\text{kN} \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= C_y - 20.250\,\text{kN} + 1.0000\,\text{kN} = 0 \\ \Rightarrow C_y &= 19.250\,\text{kN} \end{aligned}$$

$$\begin{aligned} R_C &= \sqrt{(-24.000\,\text{kN})^2 + (19.250\,\text{kN})^2} \\ &= 30.766\,\text{kN} \end{aligned}$$

C_x is negative and C_y is positive; the reaction is in the second quadrant.

$$R_{A\theta} = 180^\circ - \tan^{-1}\left[\frac{19.250}{24.000}\right] = 141.27^\circ$$



$R_A = 30.610\,\text{kN}$ at 38.367°

$B_x = -24.000\,\text{kN}$
$B_y = -1.0000\,\text{kN}$

$R_A = 30.766\,\text{kN}$ at 141.27°

Example 9: The Solution

1. Resolve the distributed loads.
2. Draw separated FBDs for the frame members.
3. Neither member is directly solvabl… any unknown. We take moment… to get an expression containing… then switch to member BCD t… another expression containing… The two equations can then be… simultaneously.
4. With B_x and B_y now known, w… x- and y-components to get th… at A and C.

From frame member BC:

$$\Sigma F_x = C_x + 24.000\,\text{kN} = 0$$
$$\Rightarrow C_x = -24.000\,\text{kN}$$

$$\Sigma F_y = C_y - 20.250\,\text{kN} + 1.0000\,\text{kN} = 0$$
$$\Rightarrow C_y = 19.250\,\text{kN}$$

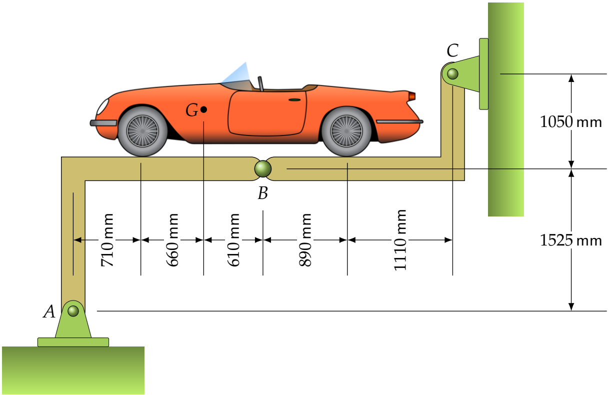
$$\ldots)^2 + (19.250\,\text{kN})^2$$

…sitive; the reaction is

$$R_{A\theta} = 180^\circ - \tan^{-1}\left[\frac{19.250}{24.000}\right] = 141.27^\circ$$

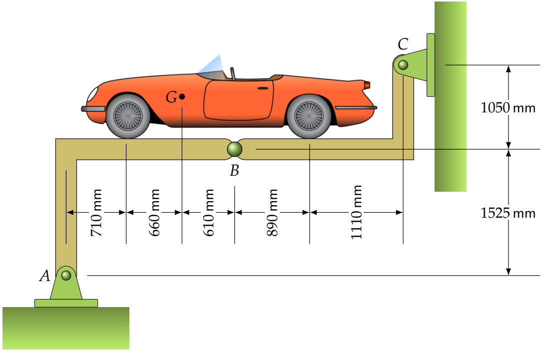
The Answers

$$R_A = 30.6\,\text{kN at } 38.4^\circ$$
$$R_C = 30.8\,\text{kN at } 141^\circ$$



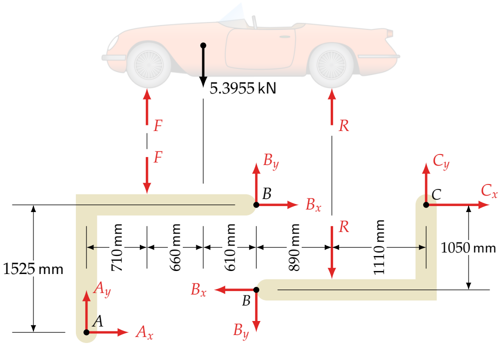
Complex Frames: Example 10

The sports car has a mass of 1100 kg and centre of gravity at G. Half of the mass is supported by the frame shown (a similar frame, hidden from view, supports the other half). All connections are pinned. Determine the reactions at A and C.



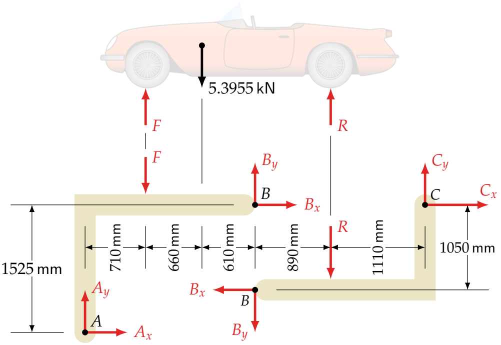
Example 10: Our Method

1. There are too many (four) unknowns to solve the entire frame at once.
2. Draw separate FBDs for each of the frame members and the car.
3. We calculate the forces exerted by the front and rear wheels on the frame.
4. Summing moments about A (for member AB) and summing moments about C (for BC) will yield two equations in the two unknowns B_x and B_y, which can then be solved.



Example 10: The Solution

1. Draw FBDs of the separated multi-force frame members.



$F = 3.7469\,\text{kN}$
$R = 1.6486\,\text{kN}$

Example 10: The Solution

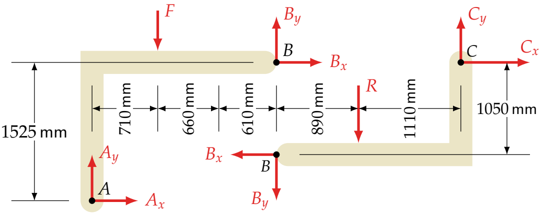
1. Draw FBDs of the separated multi-force frame members.
2. Examine the car for F and R.

Take moments about the point where the rear wheel contacts the ground:

$$\Sigma M_R = 5.3955\,\text{kN} \times 1500\,\text{mm} - F \times 2160\,\text{mm} = 0$$
$$\Rightarrow F = 3.7469\,\text{kN}$$

Then,

$$\Sigma F_y = R - 5.3955\,\text{kN} + 3.7469\,\text{kN} = 0$$
$$\Rightarrow R = 1.6486\,\text{kN}$$



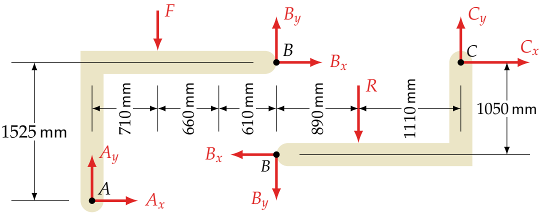
$F = 3.7469\,\text{kN}$
$R = 1.6486\,\text{kN}$

Example 10: The Solution

1. Draw FBDs of the separated multi-force frame members.
2. Examine the car for F and R.
3. Sum moments about A and C to get two equations with the two unknowns, B_x and B_y.

$$\Sigma M_A = -3.7469\,\text{kN} \times 710\,\text{mm} - B_x \times 1525\,\text{mm} + B_y \times 1980\,\text{mm} = 0$$

$$\Sigma M_C = 1.6486\,\text{kN} \times 1110\,\text{mm} + B_y \times 2000\,\text{mm} - B_x \times 1050\,\text{mm} = 0$$



$F = 3.7469$ kN
$R = 1.6486$ kN

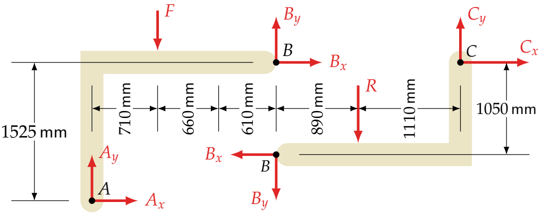
Example 10: The Solution

1. Draw FBDs of the separated multi-force frame members.
2. Examine the car for F and R.
3. Sum moments about A and C to get two equations with the two unknowns, B_x and B_y.
4. Rearrange and simplify...

$$\Sigma M_A = -3.7469\,\text{kN} \times 710\,\text{mm} - B_x \times 1525\,\text{mm} + B_y \times 1980\,\text{mm} = 0$$

$$\Sigma M_C = 1.6486\,\text{kN} \times 1110\,\text{mm} + B_y \times 2000\,\text{mm} - B_x \times 1050\,\text{mm} = 0$$

$$\begin{aligned} 1525B_x - 1980B_y &= -2660.3\,\text{kN} \\ 1050B_x - 2000B_y &= 1829.9\,\text{kN} \end{aligned}$$



$$B_x = -9.2109\,\text{kN}$$
$$B_y = -5.7507\,\text{kN}$$

$$F = 3.7469\,\text{kN}$$
$$R = 1.6486\,\text{kN}$$

Example 10: The Solution

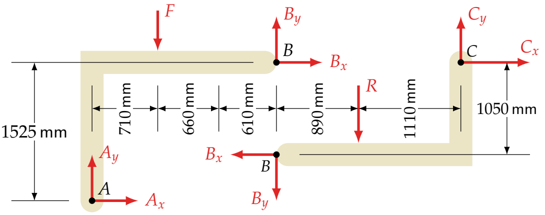
1. Draw FBDs of the separated multi-force frame members.
2. Examine the car for F and R.
3. Sum moments about A and C to get two equations with the two unknowns, B_x and B_y.
4. Rearrange and simplify...
5. Solve for B_x and B_y.

$$\begin{aligned}\Sigma M_A &= -3.7469\,\text{kN} \times 710\,\text{mm} - B_x \times 1525\,\text{mm} \\ &\quad + B_y \times 1980\,\text{mm} = 0\end{aligned}$$

$$\begin{aligned}\Sigma M_C &= 1.6486\,\text{kN} \times 1110\,\text{mm} + B_y \times 2000\,\text{mm} \\ &\quad - B_x \times 1050\,\text{mm} = 0\end{aligned}$$

$$\begin{aligned}1525B_x - 1980B_y &= -2660.3\,\text{kN} \\ 1050B_x - 2000B_y &= 1829.9\,\text{kN}\end{aligned}$$

$$B_x = -9.2109\,\text{kN}, \quad B_y = -5.7507\,\text{kN}$$



$$R_A = 13.230\,\text{kN} \text{ at } 45.878^\circ$$

$$B_x = -9.2109\,\text{kN}$$
$$B_y = -5.7507\,\text{kN}$$

$$F = 3.7469\,\text{kN}$$
$$R = 1.6486\,\text{kN}$$

Example 10: The Solution

1. Draw FBDs of the separated multi-force frame members.
2. Examine the car for F and R.
3. Sum moments about A and C to get two equations with the two unknowns, B_x and B_y.
4. Rearrange and simplify...
5. Solve for B_x and B_y.
6. Analyze member AB to find the reaction at A.

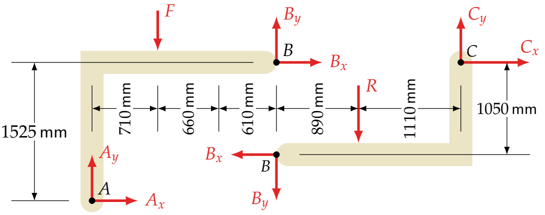
From member AB:

$$\Sigma F_x = A_x - 9.2109\,\text{kN} = 0$$
$$\Rightarrow A_x = 9.2109\,\text{kN}$$

$$\Sigma F_y = A_y - 5.7507\,\text{kN} - 3.7469\,\text{kN} = 0$$
$$\Rightarrow A_y = 9.4976\,\text{kN}$$

$$R_A = \sqrt{(9.2109\,\text{kN})^2 + (9.4976\,\text{kN})^2} = 13.230\,\text{kN}$$

$$R_{A\theta} = \tan^{-1}\left[\frac{9.4976}{9.2109}\right] = 45.878^\circ$$



$$R_A = 13.230\,\text{kN} \quad \text{at } 45.878^\circ$$

$$B_x = -9.2109\,\text{kN}$$
$$B_y = -5.7507\,\text{kN}$$

$$R_C = 10.083\,\text{kN} \quad \text{at } 204.012^\circ$$

$$F = 3.7469\,\text{kN}$$
$$R = 1.6486\,\text{kN}$$

Example 10: The Solution

1. Draw FBDs of the separated multi-force frame members.
2. Examine the car for F and R.
3. Sum moments about A and C to get two equations with the two unknowns, B_x and B_y.
4. Rearrange and simplify...
5. Solve for B_x and B_y.
6. Analyze member AB to find the reaction at A.
7. Analyze BC to find the reaction at C.

From member BC:

$$\begin{aligned}\Sigma F_x &= C_x - (-9.2109\,\text{kN}) = 0 \\ \Rightarrow C_x &= -9.2109\,\text{kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= C_y - 1.6486\,\text{kN} - (-5.7507\,\text{kN}) = 0 \\ \Rightarrow C_y &= -4.1021\,\text{kN}\end{aligned}$$

$$\begin{aligned}R_C &= \sqrt{(-9.2109\,\text{kN})^2 + (-4.1021\,\text{kN})^2} \\ &= 10.083\,\text{kN}\end{aligned}$$

$$R_{C\theta} = 180^\circ + \tan^{-1}\left[\frac{4.1021}{9.2109}\right] = 204.01^\circ$$

$R_A = 13.230\,\text{kN}$ at 45.878°

$B_x = -9.2109\,\text{kN}$
$B_y = -5.7507\,\text{kN}$

$R_C = 10.083\,\text{kN}$ at 204.012°

$F = 3.7469\,\text{kN}$
$R = 1.6486\,\text{kN}$

Example 10: Th

1. Draw FBDs of t multi-force fram
2. Examine the ca
3. Sum moments a two equations w B_x and B_y.
4. Rearrange and simplify...
5. Solve for B_x and B_y.
6. Analyze member AB to find the reaction at A.
7. Analyze BC to find the reaction at C.

The Answers

$$R_A = 13.2\,\text{kN at } 45.9^\circ$$
$$R_C = 10.1\,\text{kN at } 204^\circ$$

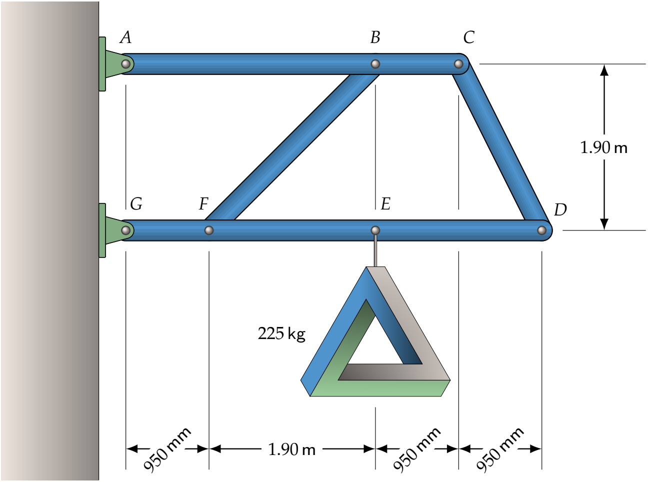
kN) = 0

) − (−5.7507 kN) = 0

$\Rightarrow C_y = -4.1021\,\text{kN}$

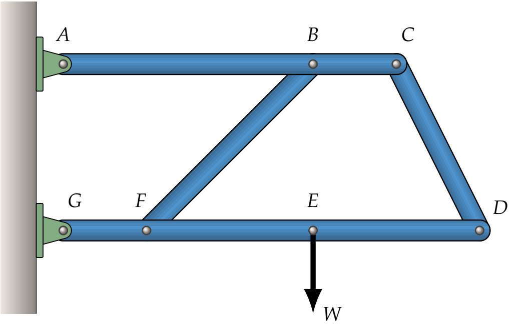
$$R_C = \sqrt{(-9.2109\,\text{kN})^2 + (-4.1021\,\text{kN})^2} = 10.083\,\text{kN}$$

$$R_{C\theta} = 180^\circ + \tan^{-1}\left[\frac{4.1021}{9.2109}\right] = 204.01^\circ$$



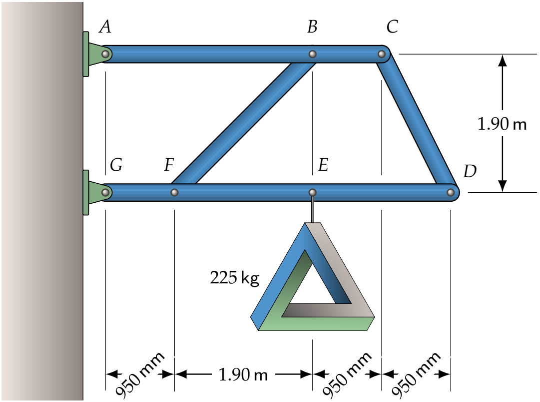
Complex Frames: Example 11

Determine the reactions at A and G,
and the forces in members BF and CD.



Example 11: Our Method

1. There are six unknown quantities: two each for the pinned supports at A and G, and the two forces in the two-force members BF and CD. There are two multi-force members, ABC and $DEFG$, that we can analyze.
2. We cannot find the supports at A and G directly so we need to separate the frame members.
3. Moments about A have two unknowns: the forces in BF and CD. Moments about G provide the same. Simultaneous equations will yield the solutions for these two internal forces.



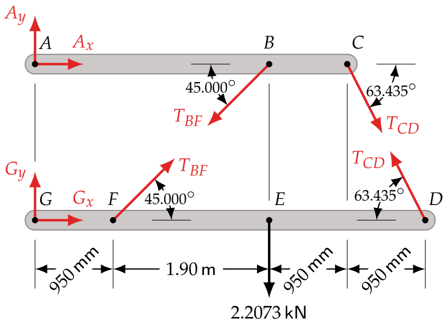
Example 11: The Solution

1. Perform preliminary calculations to find the angles between the two-force members an the multi-force members they act upon, and the weight of the mass suspended from E

$$\angle CDE = \tan^{-1}\left[\frac{1.90\,\text{m}}{0.950\,\text{m}}\right] = 63.435^\circ$$

$$\angle DFE = \tan^{-1}\left[\frac{1.90\,\text{m}}{1.90\,\text{m}}\right] = 45.000^\circ$$

$$W = 225\,\text{kg} \times 9.81\,\text{m/s}^2 = 2207.3\,\text{N}$$



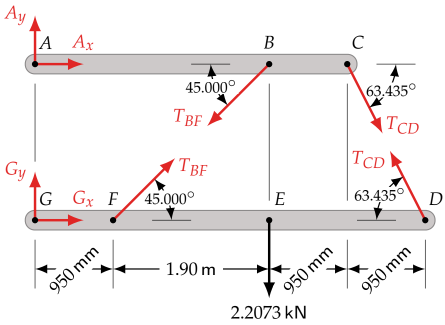
Example 11: The Solution

1. Perform preliminary calculations to find the angles between the two-force members an the multi-force members they act upon, and the weight of the mass suspended from E.
2. Draw FBDs of the separated multi-force frame members.

$$\angle CDE = \tan^{-1}\left[\frac{1.90\,\text{m}}{0.950\,\text{m}}\right] = 63.435^\circ$$

$$\angle DFE = \tan^{-1}\left[\frac{1.90\,\text{m}}{1.90\,\text{m}}\right] = 45.000^\circ$$

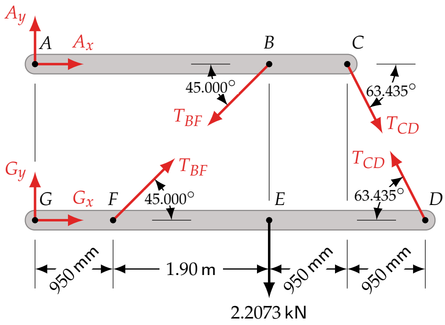
$$W = 225\,\text{kg} \times 9.81\,\text{m/s}^2 = 2207.3\,\text{N}$$



Example 11: The Solution

1. Perform preliminary calculations to find the angles between the two-force members an the multi-force members they act upon, and the weight of the mass suspended from E.
2. Draw FBDs of the separated multi-force frame members.
3. Sum moments about the pinned support connections at A and G to get two equations in the two unknowns T_{BF} and T_{CD}.

$$\begin{aligned}\Sigma M_A &= -T_{BF} \cdot \sin 45.000^\circ \times 2.8500\,\mathrm{m} \\ &\quad - T_{CD} \cdot \sin 63.435^\circ \times 3.8000\,\mathrm{m} = 0 \\ \Sigma M_G &= T_{BF} \cdot \sin 45.000^\circ \times 0.950\,\mathrm{m} \\ &\quad + T_{CD} \cdot \sin 63.435^\circ \times 4.7500\,\mathrm{m} \\ &\quad - 2.2073\,\mathrm{kN} \times 2.8500\,\mathrm{m} = 0\end{aligned}$$

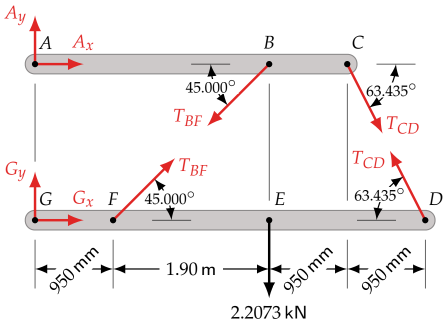


Example 11: The Solution

1. Perform preliminary calculations to find the angles between the two-force members an the multi-force members they act upon, and the weight of the mass suspended from E.
2. Draw FBDs of the separated multi-force frame members.
3. Sum moments about the pinned support connections at A and G to get two equations in the two unknowns T_{BF} and T_{CD}.
4. Simplify and rearrange.

$$\begin{aligned}\Sigma M_A &= -T_{BF}\cdot\sin 45.000^\circ\times 2.8500\,\mathrm{m}\\ &\quad - T_{CD}\cdot\sin 63.435^\circ\times 3.8000\,\mathrm{m} = 0\\ \Sigma M_G &= T_{BF}\cdot\sin 45.000^\circ\times 0.950\,\mathrm{m}\\ &\quad + T_{CD}\cdot\sin 63.435^\circ\times 4.7500\,\mathrm{m}\\ &\quad - 2.2073\,\mathrm{kN}\times 2.8500\,\mathrm{m} = 0\end{aligned}$$

$$\begin{aligned}2.0153\,T_{BF} + 3.3988\,T_{CD} &= 0\\ 0.67175\,T_{BF} + 4.2485\,T_{CD} &= 6.2908\,\mathrm{kN}\end{aligned}$$



$$T_{BF} = -3.4053\,\text{kN}$$
$$T_{CD} = 2.0191\,\text{kN}$$

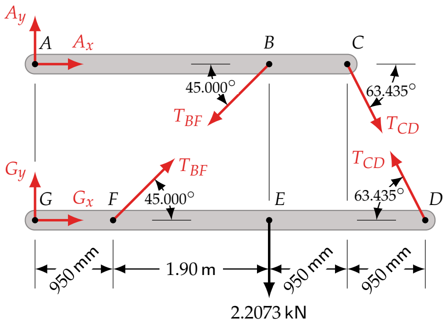
Example 11: The Solution

1. Perform preliminary calculations to find the angles between the two-force members an the multi-force members they act upon, and the weight of the mass suspended from E.
2. Draw FBDs of the separated multi-force frame members.
3. Sum moments about the pinned support connections at A and G to get two equations in the two unknowns T_{BF} and T_{CD}.
4. Simplify and rearrange.
5. Use your calculator to solve for T_{BF} and T_{CD}.

$$\begin{aligned}\Sigma M_A &= -T_{BF}\cdot \sin 45.000^\circ \times 2.8500\,\text{m}\\ &\quad - T_{CD}\cdot \sin 63.435^\circ \times 3.8000\,\text{m} = 0\\ \Sigma M_G &= T_{BF}\cdot \sin 45.000^\circ \times 0.950\,\text{m}\\ &\quad + T_{CD}\cdot \sin 63.435^\circ \times 4.7500\,\text{m}\\ &\quad - 2.2073\,\text{kN} \times 2.8500\,\text{m} = 0\end{aligned}$$

$$\begin{aligned}2.0153\,T_{BF} + 3.3988\,T_{CD} &= 0\\ 0.67175\,T_{BF} + 4.2485\,T_{CD} &= 6.2908\,\text{kN}\end{aligned}$$

$$T_{BF} = -3.4053\,\text{kN}, \quad T_{CD} = 2.0191\,\text{kN}$$



$$A_x = -3.3109\,\text{kN}$$
$$A_y = -0.60197\,\text{kN}$$

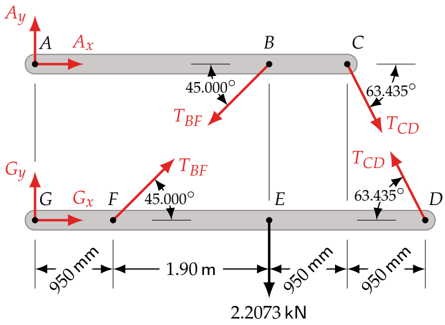
$$T_{BF} = -3.4053\,\text{kN}$$
$$T_{CD} = 2.0191\,\text{kN}$$

Example 11: The Solution

6. Calculate the reaction at A.

$$\begin{aligned}
\Sigma F_x &= A_x - T_{BF} \cdot \cos 45.000^\circ + T_{CD} \cdot \cos 63.435^\circ \\
&= A_x + 3.4053\,\text{kN} \cdot \cos 45.000^\circ \\
&\qquad + 2.0191\,\text{kN} \cdot \cos 63.435^\circ = 0 \\
\Rightarrow A_x &= -3.3109\,\text{kN}
\end{aligned}$$

$$\begin{aligned}
\Sigma F_y &= A_y - T_{BF} \cdot \sin 45.000^\circ - T_{CD} \cdot \sin 63.435^\circ \\
&= A_y + 3.4053\,\text{kN} \cdot \sin 45.000^\circ \\
&\qquad - 2.0191\,\text{kN} \cdot \sin 63.435^\circ = 0 \\
\Rightarrow A_y &= -0.60197\,\text{kN}
\end{aligned}$$



$$A_x = -3.3109\,\text{kN}$$
$$A_y = -0.60197\,\text{kN}$$
$$R_A = 3.3652\,\text{kN}$$
at 190.30°

$$T_{BF} = -3.4053\,\text{kN}$$
$$T_{CD} = 2.0191\,\text{kN}$$

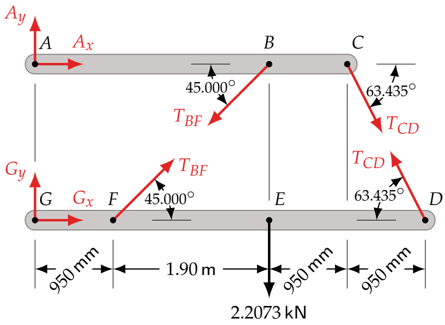
Example 11: The Solution

6. Calculate the reaction at A.

$$R_A = \sqrt{(-3.3109\,\text{kN})^2 + (-0.60197\,\text{kN})^2}$$
$$= 3.3652\,\text{kN}$$

Since both A_x and A_y are negative, the reaction is in the third quadrant:

$$R_{A\theta} = 180^\circ + \tan^{-1}\left[\frac{0.60197}{3.3109}\right] = 190.30^\circ$$



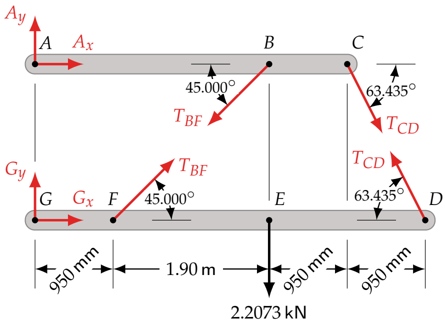
$$A_x = -3.3109\,\text{kN}$$
$$A_y = -0.60197\,\text{kN}$$
$$R_A = 3.3652\,\text{kN} \text{ at } 190.30^\circ$$
$$G_x = 3.3109\,\text{kN}$$
$$G_y = 2.8094\,\text{kN}$$
$$T_{BF} = -3.4053\,\text{kN}$$
$$T_{CD} = 2.0191\,\text{kN}$$

Example 11: The Solution

6. Calculate the reaction at A.
7. Calculate the reaction at G.

$$\begin{aligned}\Sigma F_x &= G_x + T_{BF}\cdot\cos 45.000^\circ - T_{CD}\cdot\cos 63.435^\circ\\ &= G_x - 3.4053\,\text{kN}\cdot\cos 45.000^\circ\\ &\quad - 2.0191\,\text{kN}\cdot\cos 63.435^\circ = 0\end{aligned}$$
$$\Rightarrow G_x = 3.3109\,\text{kN}$$

$$\begin{aligned}\Sigma F_y &= G_y + T_{BF}\cdot\sin 45.000^\circ\\ &\quad + T_{CD}\cdot\sin 63.435^\circ - 2.2073\,\text{kN} = 0\\ &= G_y - 3.4053\,\text{kN}\cdot\sin 45.000^\circ\\ &\quad + 2.0191\,\text{kN}\cdot\sin 63.435^\circ - 2.2073\,\text{kN} = 0\end{aligned}$$
$$\Rightarrow G_y = 2.8094\,\text{kN}$$



$$A_x = -3.3109\,\text{kN}$$
$$A_y = -0.60197\,\text{kN}$$
$$R_A = 3.3652\,\text{kN} \text{ at } 190.30^\circ$$
$$G_x = 3.3109\,\text{kN}$$
$$G_y = 2.8094\,\text{kN}$$
$$R_G = 4.3422\,\text{kN} \text{ at } 40.316^\circ$$
$$T_{BF} = -3.4053\,\text{kN}$$
$$T_{CD} = 2.0191\,\text{kN}$$

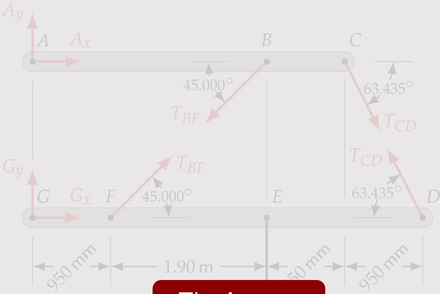
Example 11: The Solution

6. Calculate the reaction at A.
7. Calculate the reaction at G.

$$R_G = \sqrt{(-3.3109\,\text{kN})^2 + (2.8094\,\text{kN})^2} = 4.3422\,\text{kN}$$

Both G_x and G_y are positive so the reaction is in the first quadrant:

$$R_{G\theta} = \tan^{-1}\left[\frac{2.8094}{3.3109}\right] = 40.316^\circ$$



$A_x = -3.3109\,\text{kN}$
$A_y = -0.60197\,\text{kN}$
$R_A = 3.3652\,\text{kN}$ at $190.30°$

$G_x = 3.3109\,\text{kN}$
$G_y = 2.8094\,\text{kN}$

$R_G = 4.3422\,\text{kN}$ at $40.316°$

$T_{BF} = -3.4053\,\text{kN}$
$T_{CD} = 2.0191\,\text{kN}$

Example 11: The Soluti

6. Calculate the reaction at
7. Calculate the reaction at

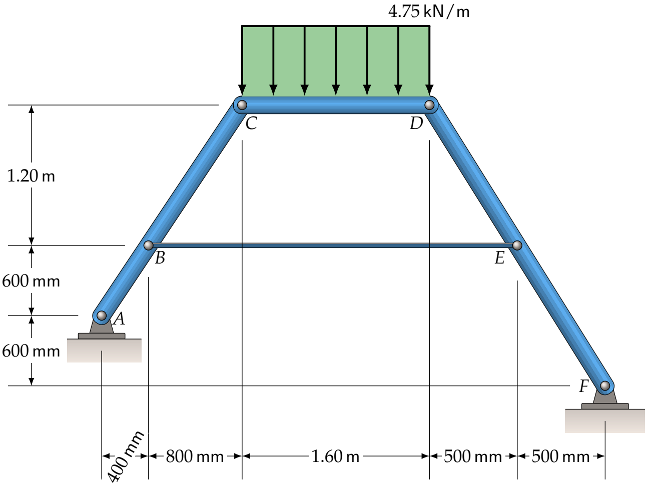
$(2.8094\,\text{kN})^2$

he reaction is in

$0.316°$

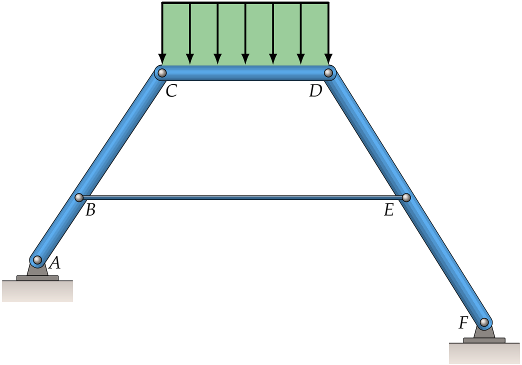
The Answers

$$R_A = 3.37\,\text{kN at } 190°$$
$$R_G = 4.34\,\text{kN at } 40.3°$$
$$T_{BF} = 3.41\,\text{kN (in compression)}$$
$$T_{CD} = 2.02\,\text{kN (in tension)}$$



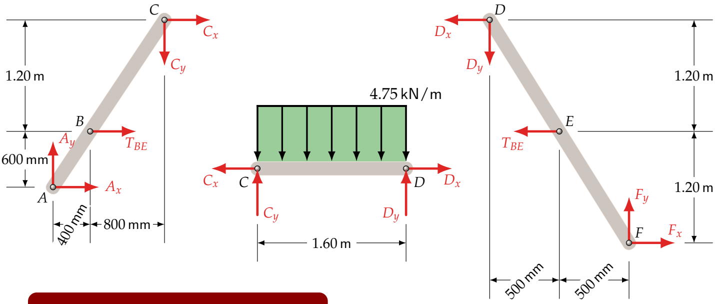
Complex Frames: Example 12

Determine the reactions at A and F,
and the tension in cable BE.



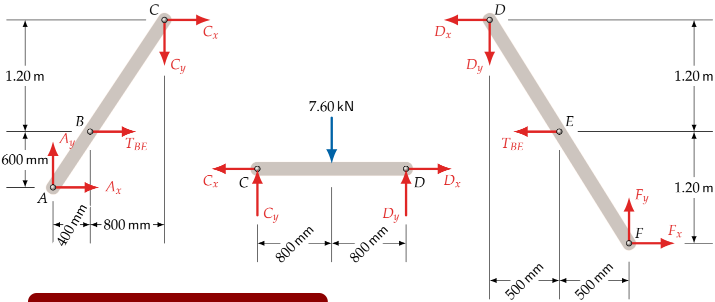
Example 12: Our Method

1. Each of the four pinned connections has two unknowns. The ninth unknown is the tension in BE. There are three frame members so the frame is statically determinant.
2. We cannot find the supports at A and F directly so we need to separate the frame members.
3. We can find the vertical components of the reactions at C and D, but not the horizontal components. The horizontal components at C and D are equal and opposite.
4. Moments about A have two unknowns: the horizontal component at C and the cable tension. Moments about F provide the same. Simultaneous equations will give the solutions.



Example 12: The Solution

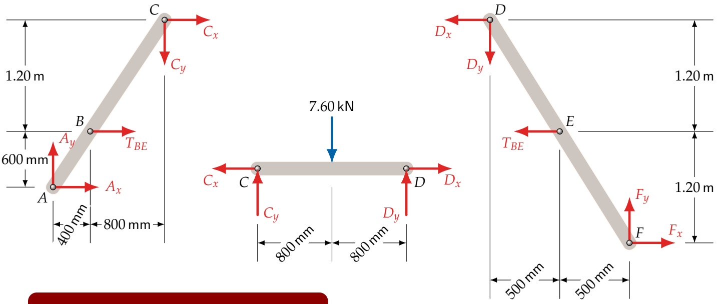
1. Separate the frame members.



Example 12: The Solution

1. Separate the frame members.
2. Resolve the distributed load.

$$4.75\,\text{kN/m} \times 1.60\,\text{m} = 7.60\,\text{kN}$$



$C_y = 3.80\,\text{kN}$

$D_y = 3.80\,\text{kN}$

Example 12: The Solution

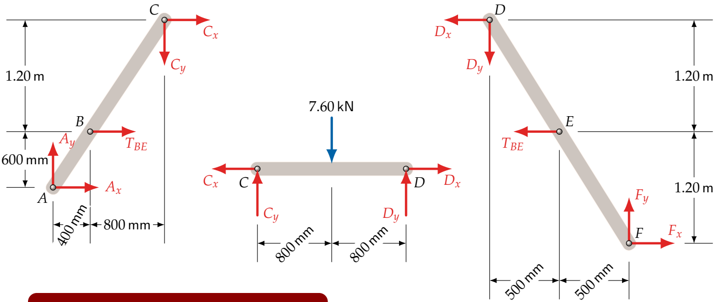
1. Separate the frame members.
2. Resolve the distributed load.
3. Analyze member CD.

$$4.75\,\text{kN/m} \times 1.60\,\text{m} = 7.60\,\text{kN}$$

$$\Sigma M_C = D_y \times 1.60\,\text{m} - 7.60\,\text{kN} \times 0.800\,\text{m} = 0$$
$$\Rightarrow D_y = 3.80\text{kN}$$

$$\Sigma F_y = C_y + 3.80\,\text{kN} - 7.60\,\text{kN} = 0$$
$$\Rightarrow C_y = 3.80\text{kN}$$

$$\Sigma F_x = -C_x + D_x = 0$$
$$\Rightarrow C_x = D_x$$



$$C_x = -4.4333\,\text{kN}$$
$$C_y = 3.80\,\text{kN}$$
$$D_x = -4.4333\,\text{kN}$$
$$D_y = 3.80\,\text{kN}$$
$$T_{BE} = 5.70\,\text{kN}$$

Example 12: The Solution

1. Separate the frame members.
2. Resolve the distributed load.
3. Analyze member CD.
4. Now, take moments about the pinned connections at A and F to get a system of two equations in the two unknowns T_{BE} and c_x (remembering that $C_x = D_x$).

$$\Sigma M_A = -C_x \cdot 1.80\,\text{m} - 3.80\,\text{kN} \cdot 1.20\,\text{m} - T_{BE} \cdot 0.600\,\text{m} = 0$$

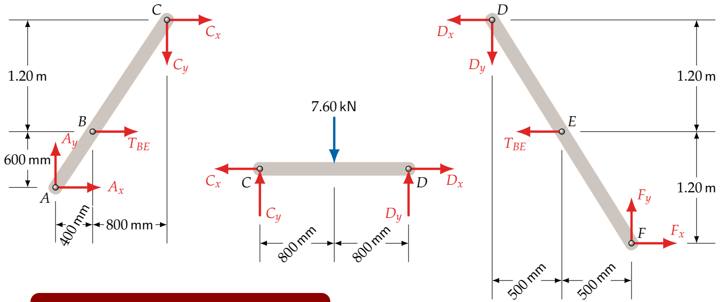
$$\Sigma M_F = D_x \cdot 2.40\,\text{m} + 3.80\,\text{kN} \cdot 1.00\,\text{m} + T_{BE} \cdot 1.20\,\text{m} = 0$$

Rearranging and simplifying the two equations:

$$1.80{\cdot}C_x + 0.600{\cdot}T_{BE} = -4.56\,\text{kN}$$
$$2.40{\cdot}C_x + 1.20{\cdot}T_{BE} = -3.80\,\text{kN}$$

From the system solver on your calculator:

$$C_x = -4.4333\,\text{kN}, \qquad T_{BE} = 5.70\,\text{kN}$$



$$R_A = 4.0056\,\text{kN at } 108.44^\circ$$

$$C_x = -4.4333\,\text{kN}$$
$$C_y = 3.80\,\text{kN}$$
$$D_x = -4.4333\,\text{kN}$$
$$D_y = 3.80\,\text{kN}$$

$$T_{BE} = 5.70\,\text{kN}$$

Example 12: The Solution

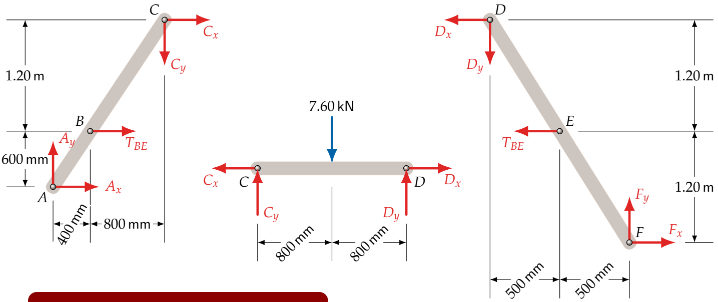
1. Separate the frame members.
2. Resolve the distributed load.
3. Analyze member CD.
4. Now, take moments about the pinned connections at A and F to get a system of two equations in the two unknowns T_{BE} and c_x (remembering that $C_x = D_x$).
5. Calculate the reaction at A.

$$\begin{aligned} \Sigma F_x &= A_x + C_x + T_{BE} \\ &= A_x - 4.4333\,\text{kN} + 5.70\,\text{kN} = 0 \\ \Rightarrow A_x &= -1.2667\,\text{kN} \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= A_y - C_y \\ &= A_y - 3.80\,\text{kN} = 0 \\ \Rightarrow A_y &= 3.80\,\text{kN} \end{aligned}$$

$$R_A = \sqrt{(-1.2667\,\text{kN})^2 + (3.80\,\text{kN})^2} = 4.0056\,\text{kN}$$

$$R_{A\theta} = 180^\circ - \tan^{-1}\left[\frac{3.80}{1.2667}\right] = 108.44^\circ$$



$R_A = 4.0056\text{ kN}$ at $108.44°$

$C_x = -4.4333\text{ kN}$

$C_y = 3.80\text{ kN}$

$D_x = -4.4333\text{ kN}$

$D_y = 3.80\text{ kN}$

$R_F = 4.0056\text{ kN}$ at $71.565°$

$T_{BE} = 5.70\text{ kN}$

Example 12: The Solution

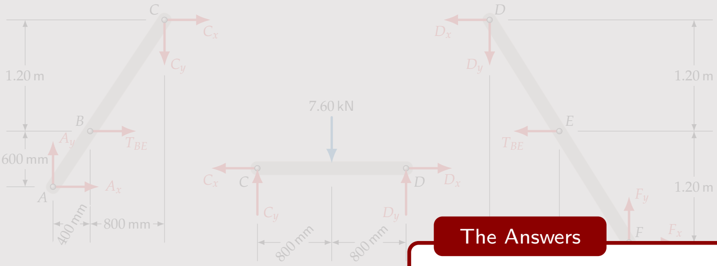
1. Separate the frame members.
2. Resolve the distributed load.
3. Analyze member CD.
4. Now, take moments about the pinned connections at A and F to get a system of two equations in the two unknowns T_{BE} and c_x (remembering that $C_x = D_x$).
5. Calculate the reaction at A.
6. Calculate the reaction at F.

$$\begin{aligned}\Sigma F_x &= F_x - D_x - T_{BE} \\ &= F_x + 4.4333\text{ kN} - 5.70\text{ kN} = 0 \\ \Rightarrow F_x &= 1.2667\text{ kN}\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= F_y - D_y \\ &= F_y - 3.80\text{ kN} = 0 \\ \Rightarrow F_y &= 3.80\text{ kN}\end{aligned}$$

$$R_A = \sqrt{(1.2667\text{ kN})^2 + (3.80\text{ kN})^2} = 4.0056\text{ kN}$$

$$R_{A\theta} = \tan^{-1}\left[\frac{3.80}{1.2667}\right] = 71.565°$$



$R_A = 4.0056\,\text{kN}$ at 108.44°

$C_x = -4.4333\,\text{kN}$

$C_y = 3.80\,\text{kN}$

$D_x = -4.4333\,\text{kN}$

$D_y = 3.80\,\text{kN}$

$R_F = 4.0056\,\text{kN}$ at 71.565°

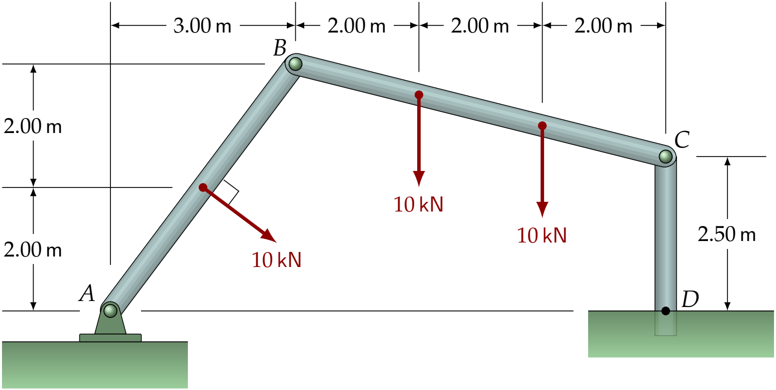
70 kN

The Answers

$$R_A = 4.01\,\text{kN at } 108^\circ$$
$$R_F = 4.01\,\text{kN at } 71.6^\circ$$
$$T_{BE} = 5.70\,\text{kN}$$

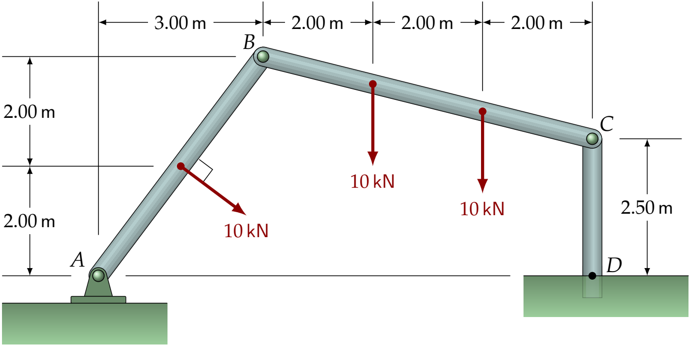
Example 12: The Solution

1. Separate the frame members.
2. Resolve the distributed load.
3. Analyze member CD.
4. Now, take moments about the pinned connections at A and F to get a system of two equations in the two unknowns T_{BE} and c_x (remembering that $C_x = D_x$).
5. Calculate the reaction at A.
6. Calculate the reaction at F.



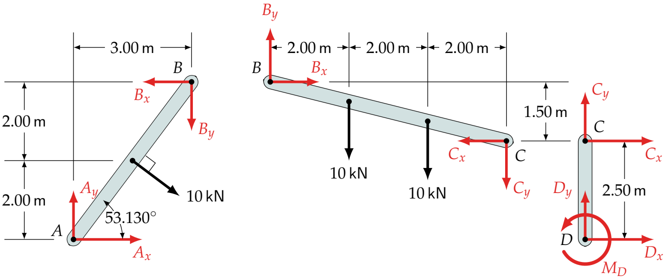
Complex Frames: Example 13

A, B and C are pinned connections. D is a fixed connection. Determine the reactions at A and D.



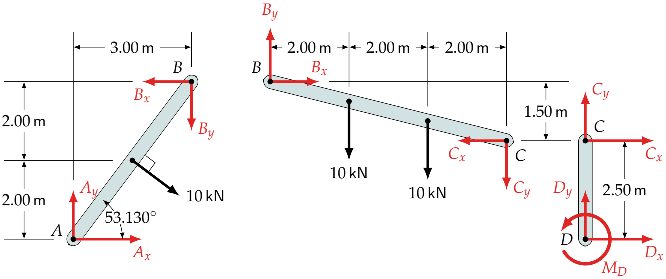
Example 13: Our Method

1. A and D have 5 unknowns between them so we cannot start with the support reactions.
2. We do not (yet) know the reacting moment at D so summing moments about B (for member BC) and moments about D (for member CD) will have two equations and the three unknowns C_x, C_y and M_D.
3. Summing moments about A (for member AB) and summing moments about C (for BC) will yield two equations in the two unknowns B_x and B_y, which can be solved.



Example 13: The Solution

1. Draw FBDs of the separated multi-force frame members.

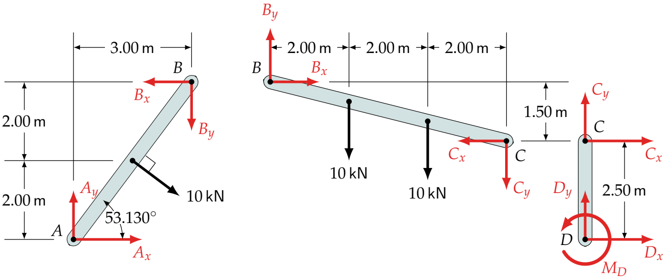


Example 13: The Solution

1. Draw FBDs of the separated multi-force frame members.
2. Sum moments about A and C to find equations including B_x and B_y.

$$|AB| = \sqrt{(3.00\,\text{m})^2 + (4.00\,\text{m})^2} = 5.0000\,\text{m}$$

$$\begin{aligned}\Sigma M_A &= B_x \times 4.00\,\text{m} - B_y \times 3.00\,\text{m} \\ &\quad - 10\,\text{kN} \times 5.0000\,\text{m}/2 = 0 \\ \Sigma M_C &= 10\,\text{kN} \times 2.00\,\text{m} + 10\,\text{kN} \times 4.00\,\text{m} \\ &\quad - B_x \times 1.50\,\text{m} - B_y \times 6.00\,\text{m} = 0\end{aligned}$$



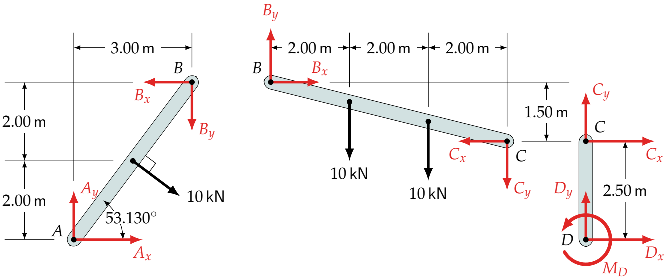
Example 13: The Solution

1. Draw FBDs of the separated multi-force frame members.
2. Sum moments about A and C to find equations including B_x and B_y.
3. Rearrange and simplify...

$$|AB| = \sqrt{(3.00\,\text{m})^2 + (4.00\,\text{m})^2} = 5.0000\,\text{m}$$

$$\begin{aligned}\Sigma M_A &= B_x \times 4.00\,\text{m} - B_y \times 3.00\,\text{m} \\ &\quad - 10\,\text{kN} \times 5.0000\,\text{m}/2 = 0 \\ \Sigma M_C &= 10\,\text{kN} \times 2.00\,\text{m} + 10\,\text{kN} \times 4.00\,\text{m} \\ &\quad - B_x \times 1.50\,\text{m} - B_y \times 6.00\,\text{m} = 0\end{aligned}$$

$$\begin{aligned}4.00\,B_x - 3.00\,B_y &= 25.000\,\text{kN} \\ 1.50\,B_x + 6.00\,B_y &= 60.0\,\text{kN}\end{aligned}$$



$$B_x = 11.579\,\text{kN}$$
$$B_y = 7.1053\,\text{kN}$$

Example 13: The Solution

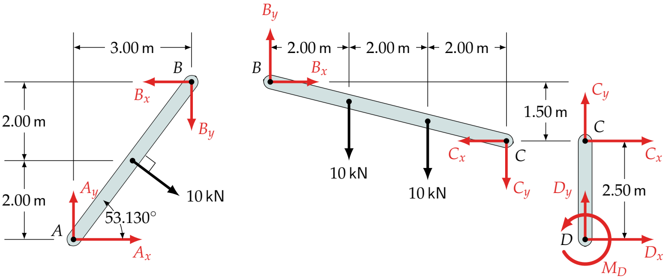
1. Draw FBDs of the separated multi-force frame members.
2. Sum moments about A and C to find equations including B_x and B_y.
3. Rearrange and simplify...
4. Solve for B_x and B_y.

$$|AB| = \sqrt{(3.00\,\text{m})^2 + (4.00\,\text{m})^2} = 5.0000\,\text{m}$$

$$\begin{aligned}\Sigma M_A &= B_x \times 4.00\,\text{m} - B_y \times 3.00\,\text{m} \\ &\quad - 10\,\text{kN} \times 5.0000\,\text{m}/2 = 0 \\ \Sigma M_C &= 10\,\text{kN} \times 2.00\,\text{m} + 10\,\text{kN} \times 4.00\,\text{m} \\ &\quad - B_x \times 1.50\,\text{m} - B_y \times 6.00\,\text{m} = 0\end{aligned}$$

$$\begin{aligned}4.00\,B_x - 3.00\,B_y &= 25.000\,\text{kN} \\ 1.50\,B_x + 6.00\,B_y &= 60.0\,\text{kN}\end{aligned}$$

$$B_x = 11.579\,\text{kN}, \qquad B_y = 7.1053\,\text{kN}$$



$$B_x = 11.579\,\text{kN}$$
$$B_y = 7.1053\,\text{kN}$$

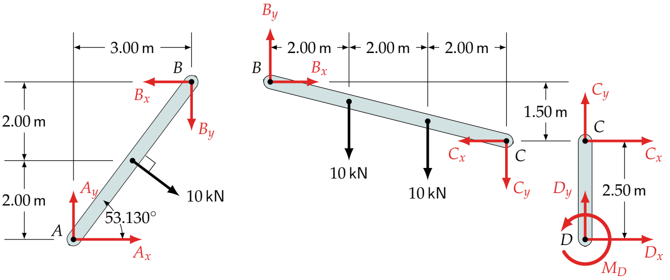
$$C_x = 11.579\,\text{kN}$$
$$C_y = -12.895\,\text{kN}$$

Example 13: The Solution

1. Draw FBDs of the separated multi-force frame members.
2. Sum moments about A and C to find equations including B_x and B_y.
3. Rearrange and simplify...
4. Solve for B_x and B_y.
5. Analyze member BC to find C_x and C_y.

$$\Sigma F_x = B_x - C_x = 0$$
$$\Rightarrow C_x = B_x = 11.579\,\text{kN}$$

$$\Sigma F_y = B_y - C_y - 20\,\text{kN} = 0$$
$$\Rightarrow C_y = -12.895\,\text{kN}$$



$$
\begin{aligned}
B_x &= 11.579\,\text{kN}\\
B_y &= 7.1053\,\text{kN}\\[1ex]
C_x &= 11.579\,\text{kN}\\
C_y &= -12.895\,\text{kN}\\[1ex]
R_D &= 17.331\,\text{kN}\\
&\qquad \text{at } 131.92^\circ\\
M_D &= 28.948\,\text{kN}\cdot\text{m}
\end{aligned}
$$

Example 13: The Solution

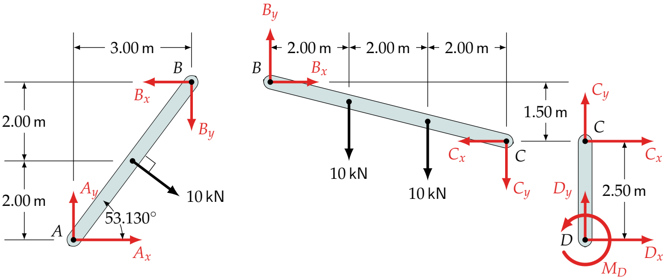
1. Draw FBDs of the separated multi-force frame members.
2. Sum moments about A and C to find equations including B_x and B_y.
3. Rearrange and simplify...
4. Solve for B_x and B_y.
5. Analyze member BC to find C_x and C_y.
6. Analyze member CD to find the reaction and reacting moment at D.

$$
\begin{aligned}
\Sigma M_D &= M_D - 11.579\,\text{kN} \times 2.50\,\text{m} = 0\\
\Rightarrow M_D &= 28.948\,\text{kN}\cdot\text{m}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x &= D_x + 11.579\,\text{kN} = 0\\
\Rightarrow D_x &= -11.579\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_y &= D_y + (-12.895\,\text{kN}) = 0\\
\Rightarrow D_y &= 12.895\,\text{kN}
\end{aligned}
$$

$$
\begin{aligned}
R_D &= \sqrt{(-11.579\,\text{kN})^2 + (12.895\,\text{kN})^2}\\
&= 17.331\,\text{kN}\\
R_{D\theta} &= 180^\circ - \tan^{-1}\left[\frac{12.895}{11.579}\right] = 131.92^\circ
\end{aligned}
$$



$$R_A = 13.585\,\text{kN} \quad \text{at } 74.725^\circ$$

$$B_x = 11.579\,\text{kN}$$
$$B_y = 7.1053\,\text{kN}$$

$$C_x = 11.579\,\text{kN}$$
$$C_y = -12.895\,\text{kN}$$

$$R_D = 17.331\,\text{kN} \quad \text{at } 131.92^\circ$$
$$M_D = 28.948\,\text{kN}\cdot\text{m}$$

Example 13: The Solution

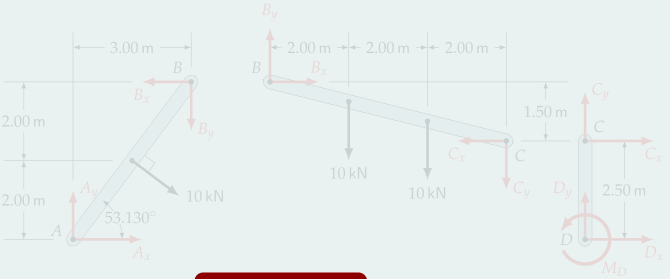
1. Draw FBDs of the separated multi-force frame members.
2. Sum moments about A and C to find equations including B_x and B_y.
3. Rearrange and simplify...
4. Solve for B_x and B_y.
5. Analyze member BC to find C_x and C_y.
6. Analyze member CD to find the reaction and reacting moment at D.
7. Analyze AB to find the reaction at A.

$$\Sigma F_x = A_x + 10\,\text{kN}\cdot\sin 53.130^\circ - 11.579\,\text{kN} = 0$$
$$\Rightarrow A_x = 3.5790\,\text{kN}$$

$$\Sigma F_y = A_y - 10\,\text{kN}\cdot\cos 53.130^\circ - 7.1053\,\text{kN} = 0$$
$$\Rightarrow A_y = 13.105\,\text{kN}$$

$$R_A = \sqrt{(3.5790\,\text{kN})^2 + (13.105\,\text{kN})^2} = 13.585\,\text{kN}$$

$$R_{A\theta} = \tan^{-1}\left[\frac{13.105}{3.5790}\right] = 74.725^\circ$$



$R_A = 13.585\,\text{kN}$ at 74.725°

$B_x = 11.579\,\text{kN}$
$B_y = 7.1053\,\text{kN}$

$C_x = 11.579\,\text{kN}$
$C_y = -12.895\,\text{kN}$

$R_D = 17.331\,\text{kN}$ at 131.92°
$M_D = 28.948\,\text{kN}\cdot\text{m}$

Example 13:

1. Draw FBDs of t… frame members…
2. Sum moments a… equations incluc…
3. Rearrange and s…
4. Solve for B_x and B_y.
5. Analyze member BC to find C_x and C_y.
6. Analyze member CD to find the reaction and reacting moment at D.
7. Analyze AB to find the reaction at A.

The Answers

$$R_A = 13.6\,\text{kN at } 74.7^\circ$$
$$R_D = 17.3\,\text{kN at } 132^\circ$$
$$M_D = 28.9\,\text{kN}\cdot\text{m}$$