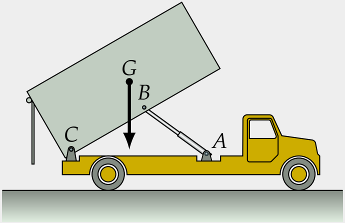
09 Complex Frames

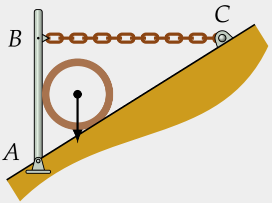
Engineering Statics

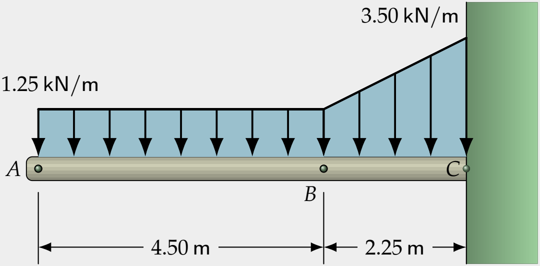
Updated on: November 20, 2025

Simple Frames

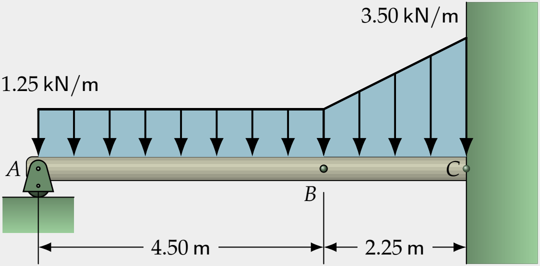
- In the problems we investigated in the module on the equilibrium of rigid bodies, there was a structural member, acted upon by a force (or forces), each with a known magnitude and direction (such as its weight and/or applied loads).
- There was a single force with a known direction but unknown magnitude (such as a hydraulic hoist, or a chain in tension,...) and a reaction with unknown x- and y-components.
- We are limited, by the equations of equilibrium, to solving for a maximum of three unknowns. These problems can be considered **simple frames**.







There is a fixed connection at C. We have solved problems like this.



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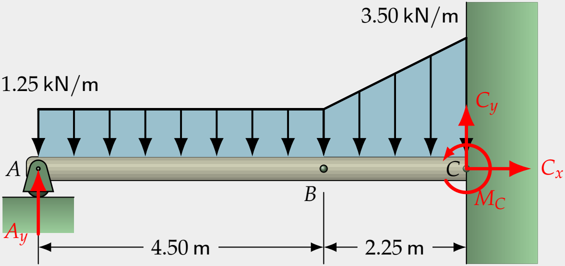
But how about now? How many unknowns are there?



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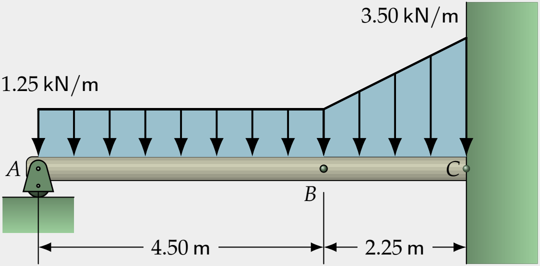

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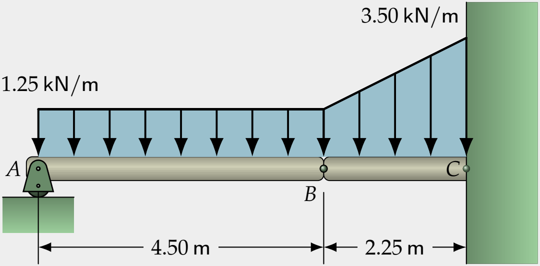
With the three equations of statics, we can only solve for three unknowns. We cannot solve this with statics alone. This is a **statically indeterminant** problem.

Statical Determinancy



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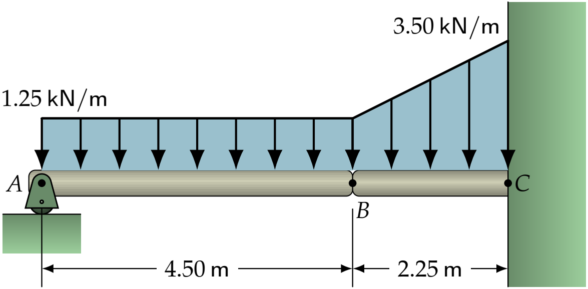
But how about now? How many unknowns are there? **Four!**

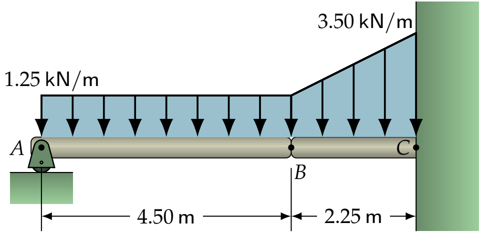
With the three equations of statics, we can only solve for three unknowns. We cannot solve this with statics alone. This is a **statically indeterminant** problem.

But if there is a pinned connection along AB, we increase the number of members. This becomes a complex frame and we **can** solve it!

Example 1

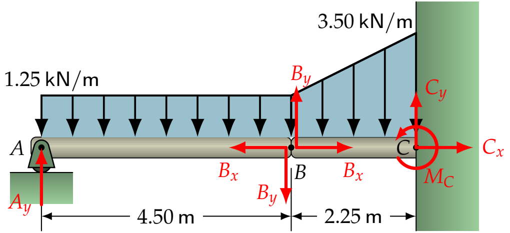


There is a roller at A, a pinned connection at B and a fixed connection at C. Determine the reactions at A and C.



Example 1: Solution

1. How many unknowns are there?

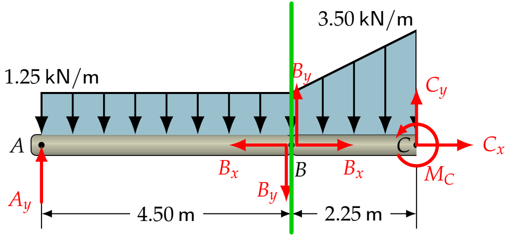


Example 1: Solution

1. How many unknowns are there?

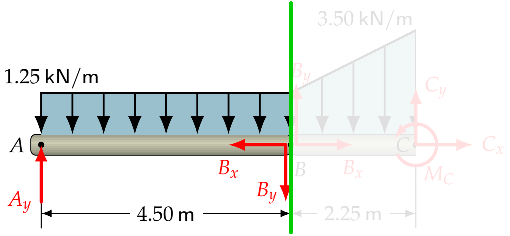
   Notice that member AB exerts an equal and opposite force (with components B_x and B_y) on member BC. This is a necessary condition for equilibrium at B.

   A_y, B_x, B_y, C_x, C_y and M_C makes 6 unknowns. But now we have two members, so we can write 6 equations of equilibrium (3 for each member). We can solve this problem.



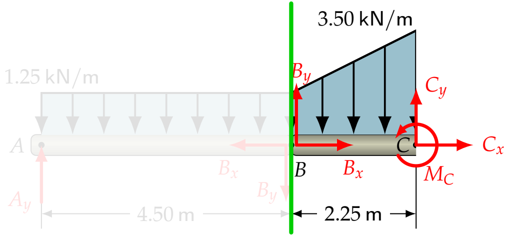
Example 1: Our Method

- Consider a vertical section through B.



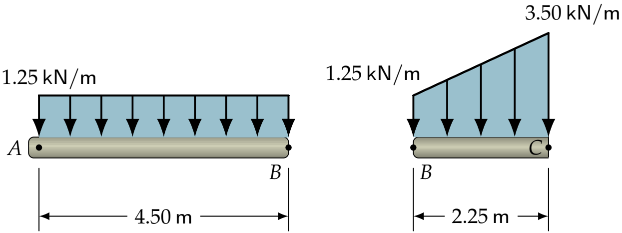
Example 1: Our Method

- Consider a vertical section through B.
- The portion to the left of the section (that is, member AB) is in equilibrium. It has three unknowns: A_y, B_x and B_y. We can solve for these.



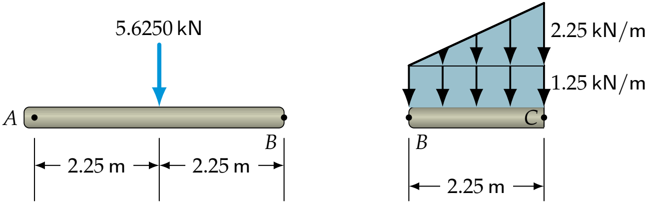
Example 1: Our Method

- Consider a vertical section through B.
- The portion to the left of the section (that is, member AB) is in equilibrium. It has three unknowns: A_y, B_x and B_y. We can solve for these.
- The portion to the right of the section (member BC), now that we know B_x and B_y, has three remaining unknowns: M_C, C_x and C_y. We can solve for these.



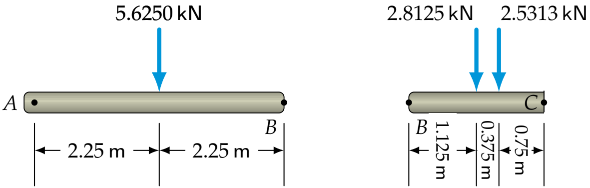
Example 1: Solution

2. Draw members separated for more convenient analysis.



Example 1: Solution

2. Draw members separated for more convenient analysis.
3. Resolve the distributed loads.



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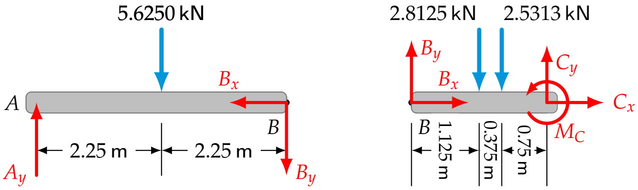

Example 1: Solution

2. Draw members separated for more convenient analysis.
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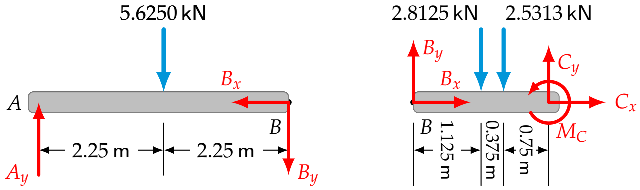
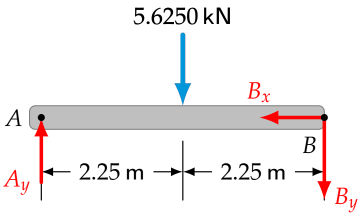
4. Complete the free-body diagrams.



Example 1: Solution

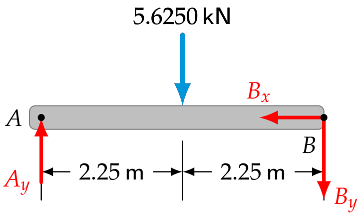
2. Draw members separated for more convenient analysis.
3. Resolve the distributed loads.
4. Complete the free-body diagrams.
5. Now, analyze member AB for A_y, B_x and B_y.



$A_y = 2.8125\,\text{kN}$

Example 1: Solve AB

$$\Sigma M_B = (5.6250\,\text{kN})\cdot(2.25\,\text{m}) - A_y(4.50\,\text{m}) = 0$$
$$\Rightarrow A_y = 2.8125\,\text{kN}$$



$$A_y = 2.8125\,\text{kN}$$
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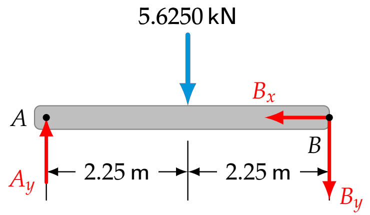

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$$B_x = 0$$

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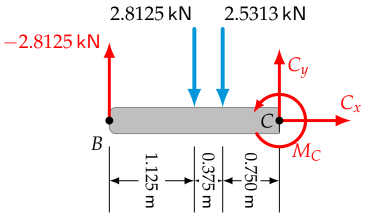
$$\Sigma F_x = B_x = 0$$

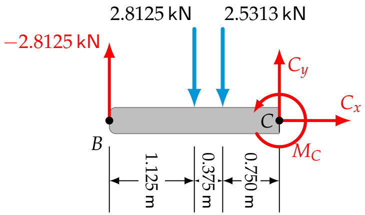


$$A_y = 2.8125\,\text{kN}$$
$$B_y = -2.8125\,\text{kN}$$
$$B_x = 0$$
$$M_C = -11.391\,\text{kN}\cdot\text{m}$$

Example 1: Solve BC

$$\Sigma M_C = M_C + (2.5313\,\text{kN})\cdot(0.750\,\text{m}) + (2.8125\,\text{kN})\cdot(1.125\,\text{m}) - (-2.8125\,\text{kN})\cdot(2.25\,\text{m}) = 0$$

$$\Rightarrow M_C = 11.391\,\text{kN}\cdot\text{m}$$



$$
\begin{aligned}
A_y &= 2.8125\,\text{kN} \\
B_y &= -2.8125\,\text{kN} \\
B_x &= 0 \\
M_C &= -11.391\,\text{kN}\cdot\text{m} \\
C_y &= 8.5163\,\text{kN}
\end{aligned}
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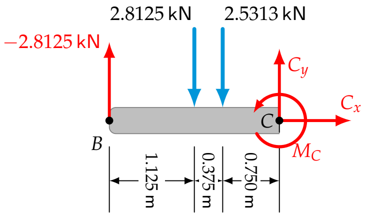

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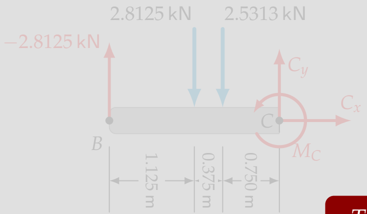

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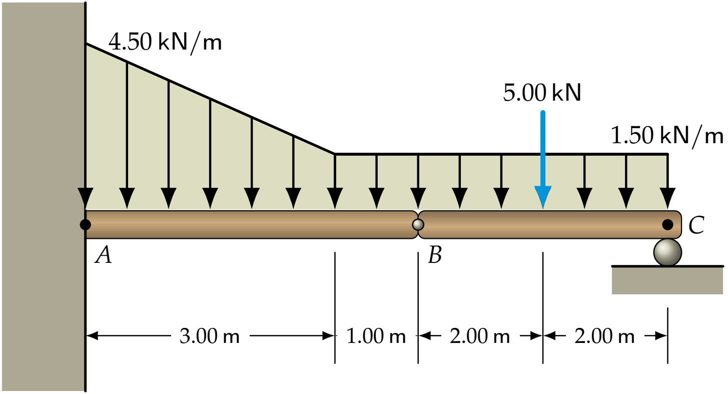
$$\Rightarrow C_y = 8.5163\,\text{kN}$$

$$\Sigma F_x = C_x = 0$$

The Answers

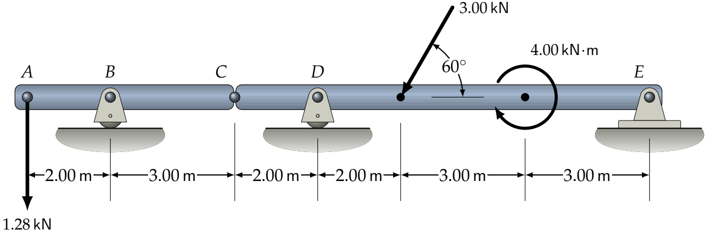
$$R_A = 2.81\,\text{kN at } 90^\circ$$
$$R_C = 8.52\,\text{kN at } 90^\circ$$
$$M_C = -11.4\,\text{kN}\cdot\text{m}$$

Exercise 1



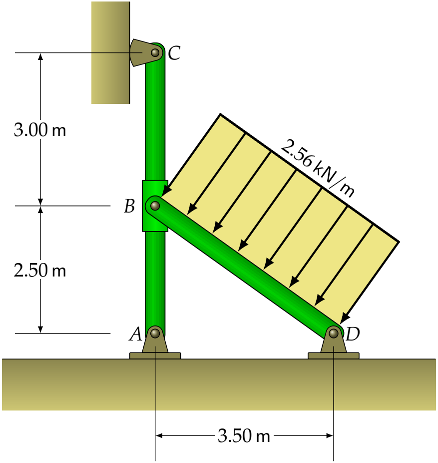
There is a fixed connection at A, a pinned connection at B and a roller at C. Determine the reactions at A and C.

Example 2



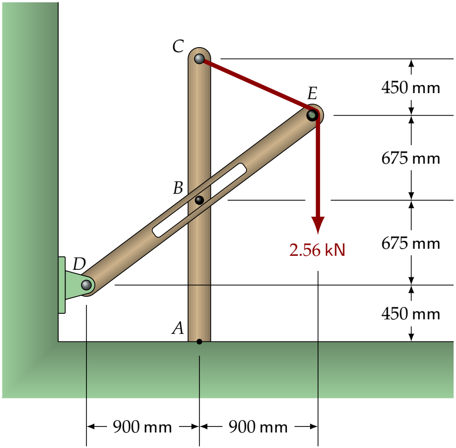
There are frictionless rollers at B and D and a pinned connection at E. Determine the reactions at B, D and$E$.

Example 3



There is a smooth collar at B, a rocker at C and pinned connections at A and D. Determine the force that the collar at B exerts on member BD, and the reactions at A and D.

Example 4



There are smooth pegs B and E, and a pinned connection at D. A is fixed connection. Determine the reactions at A and D.

Increasing complexity...

- In the 'complex' frames that we have examined so far, we have been able to immediately determine all the forces acting on one of the members:

  Example 1: We could determine all the forces acting on member AB, and use those results to determine the forces acting on BC.

  Example 2: We determined the forces acting on member AC, and used those results to determine the forces acting on CE.

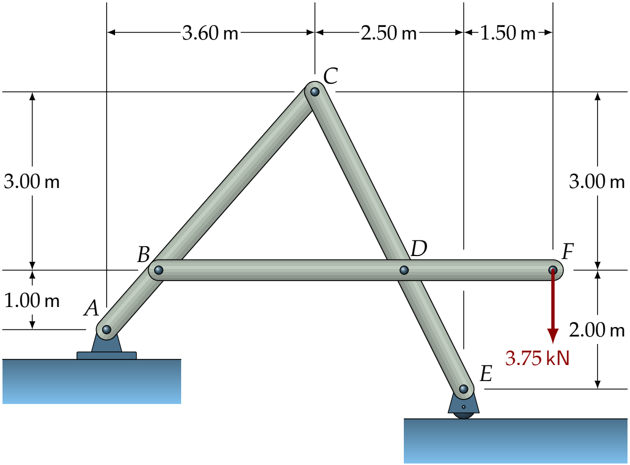
  Example 3: Similarly, examining member BD reduced the number of unknown forces acting on AC.

  Example 4: We employed a similar process again...

- Not all frames can be solved this way.

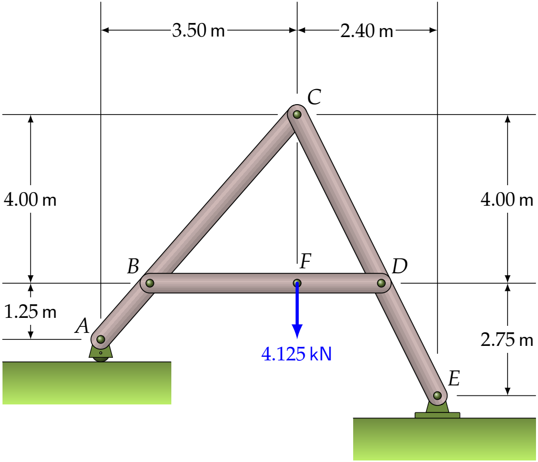
  If we can solve for the external supports (that is, with three supprt unknowns such as a pinned connection and a rocker/roller), that often reduces the difficulty of the problem. The next examples demonstrate this.

Example 5



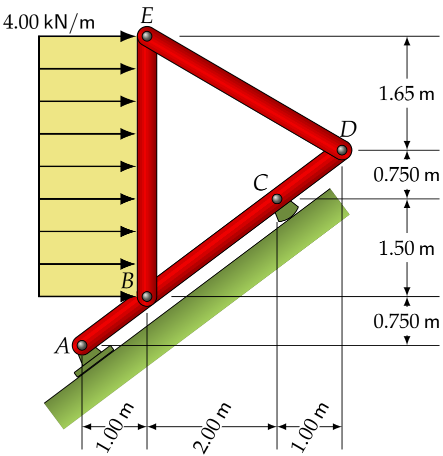
Determine the magnitude of the reactions at A, B and C.

Exercise 2



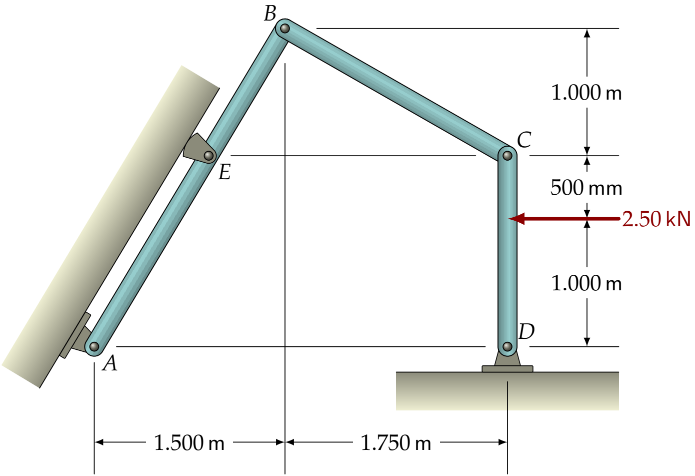
Determine the magnitude of the reactions at A, B and C

Example 6



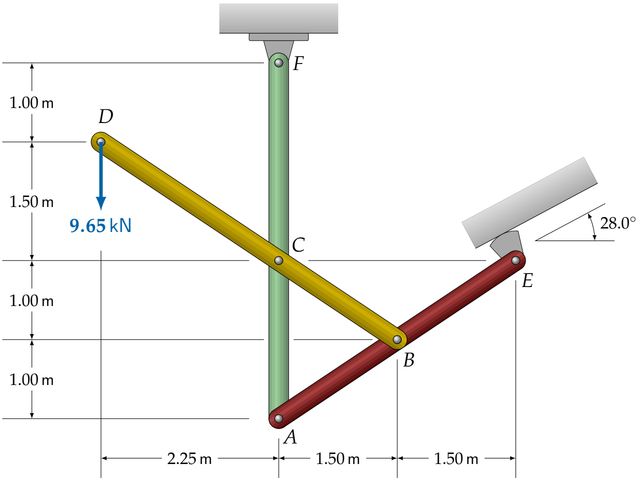
Determine the reactions at A and C, and the internal force in two-force member DE.

Exercise 3



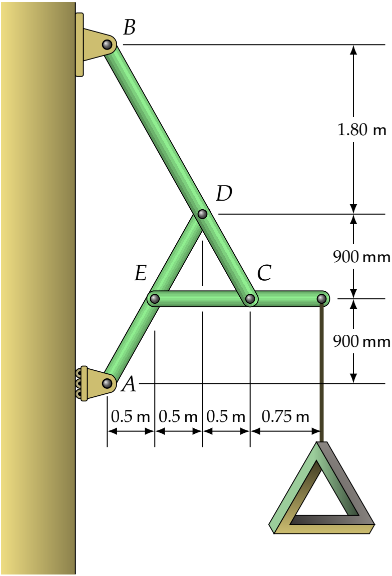
Determine the reactions at A and C, and the internal force in two-force member DE.

Example 7



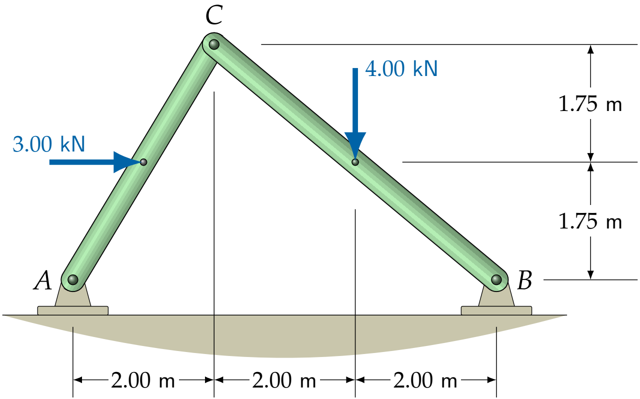
Determine the magnitude of reactions in A, B and C due to the 9.65 kN load.

Exercise 4

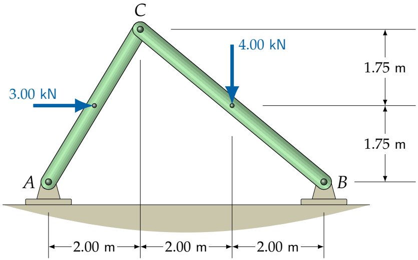


The suspended sign has a mass of 112 kg. Determine the magnitudes of the reactions at the pinned connections C, D and E.

Example 8

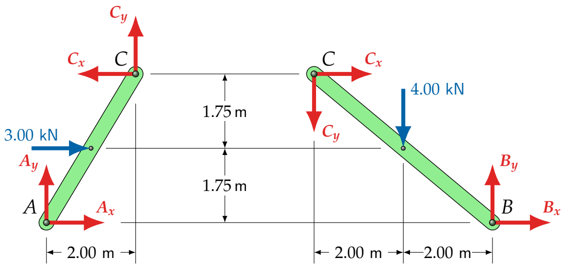


Determine the components of the reactions at A, B and C.



Example 8: Solution

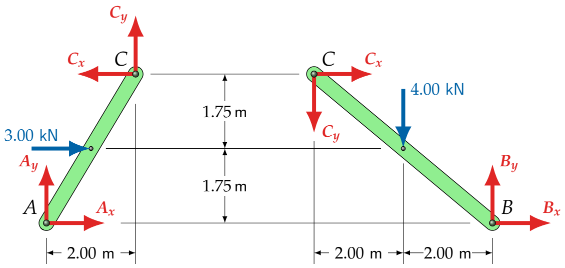
1. The pinned connections at A and B each have two unknown reaction components so we cannot do anything 'globally' with the whole frame. Separate the two frame members and draw the free-body diagrams.



Example 8: Solution

2. Consider member AC:

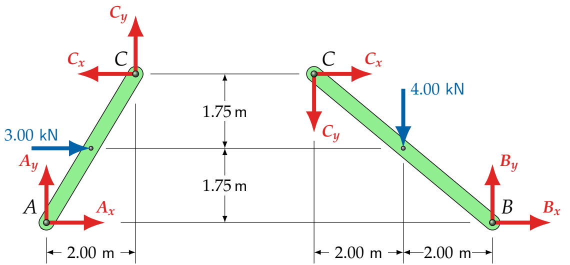
- C_x and C_y are the components of the force exerted on C by the other frame member, BC
- It is unimportant in which direction they are drawn.
- As usual, both components of the reaction at A are drawn in the positive direction.
- There are too many unknowns (4) to solve this member alone.



Example 8: Solution

3. Now consider member BC:

- C_x and C_y **must** be drawn in the **opposite** direction to those drawn for member AC to satisfy equilibrium
- As usual, both components of the reaction at A are drawn in the positive direction.
- There are too many unknowns (4) to solve this member alone.
- However, we now have 6 unknowns from the two members so we can solve this frame for the required reactions. How?

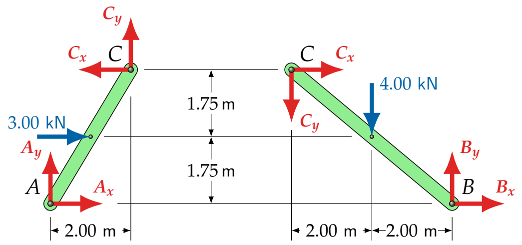


Example 8: Solution

4. Taking moments about the pinned connection at A will give an expression with two unknowns in it: C_x and C_y.

   Similarly, taking moments about the pinned connection at B will give a second expression with two unknowns in it: C_x and C_y.

   These two expressions can be solved simultaneously to give C_x and C_y.



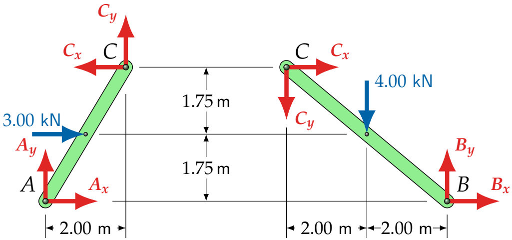
$$C_x = 1.7619\,\text{kN}$$
$$C_y = -0.45833\,\text{kN}$$

Example 8: Solution

$$\Sigma M_A = 0 = (3.50\,\text{m})C_x + (2.00\,\text{m})C_y - (1.75\,\text{m})(3.00\,\text{kN}) \quad (1)$$
$$\Sigma M_B = 0 = (-3.50\,\text{m})C_x + (4.00\,\text{m})C_y + (2.00\,\text{m})(4.00\,\text{kN}) \quad (2)$$

Adding (1) and (2) (or using your calculator system-solver) gives :

$$0 = (6.00\,\text{m})C_y + 2.75\,\text{kN}\cdot\text{m}$$
$$\Rightarrow C_y = -0.45833\,\text{kN}$$
$$\Rightarrow C_x = 1.7619\,\text{kN}$$



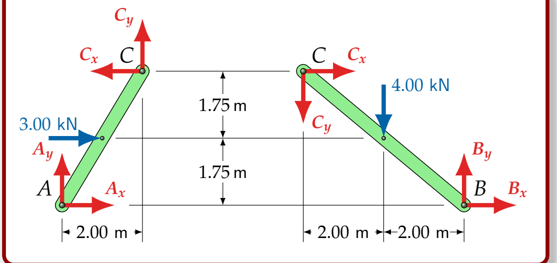
$$A_x = -1.2381\,\text{kN}$$
$$A_y = 0.45833\,\text{kN}$$
$$C_x = 1.7619\,\text{kN}$$
$$C_y = -0.45833\,\text{kN}$$

Example 8: Solution

Reactions at A:

$$\Sigma F_x = 0 = A_x + 3.00\,\text{kN} - 1.7619\,\text{kN}$$
$$\Rightarrow A_x = -1.2381\,\text{kN}$$

$$\Sigma F_y = 0 = A_y - 0.45833\,\text{kN}$$
$$\Rightarrow A_y = 0.45833\,\text{kN}$$



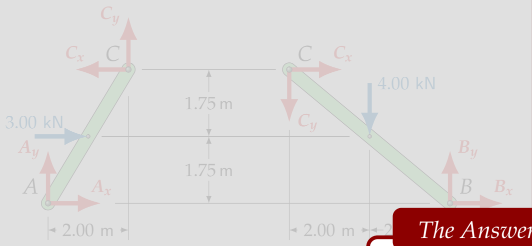
$$A_x = -1.2381\,\text{kN}$$
$$A_y = 0.45833\,\text{kN}$$
$$B_x = -1.7619\,\text{kN}$$
$$B_y = 3.5417\,\text{kN}$$
$$C_x = 1.7619\,\text{kN}$$
$$C_y = -0.45833\,\text{kN}$$

Example 8: Solution

Reactions at B:

$$\Sigma F_x = 0 = B_x + 1.7619\,\text{kN}$$
$$\Rightarrow B_x = -1.7619\,\text{kN}$$

$$\Sigma F_y = 0 = B_y + 0.45833\,\text{kN} - 4.000\ \text{m}$$
$$\Rightarrow B_y = 3.5417\,\text{kN}$$



$$A_x = -1.2381\,\text{kN}$$
$$A_y = 0.45833\,\text{kN}$$
$$B_x = -1.7619\,\text{kN}$$
$$B_y = 3.5417\,\text{kN}$$
$$C_x = 1.7619\,\text{kN}$$
$$C_y = -0.45833\,\text{kN}$$

Example 8: Solution

Reactions at B:

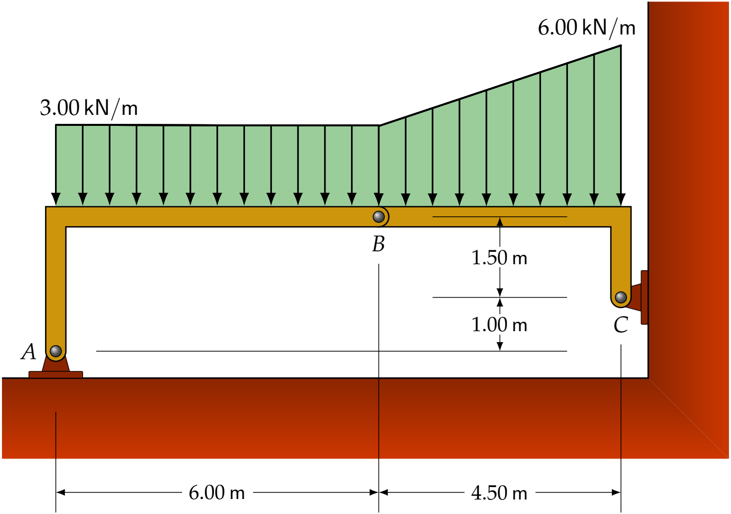
$$\Sigma F_x = 0 = B_x + \ldots$$
$$\Rightarrow B_x = -1.7\ldots$$

$$\Sigma F_y = 0 = B_y + \ldots$$
$$\Rightarrow B_y = 3.541\ldots$$

The Answers

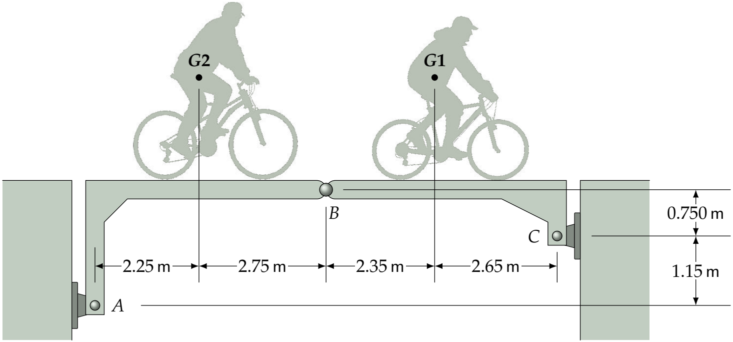
$$A_x = -1.24\,\text{kN}$$
$$A_y = 0.458\,\text{kN}$$
$$B_x = -1.76\,\text{kN}$$
$$B_y = 3.54\,\text{kN}$$
$$C_x = 1.76\,\text{kN}$$
$$C_y = -0.458\,\text{kN}$$

Example 9



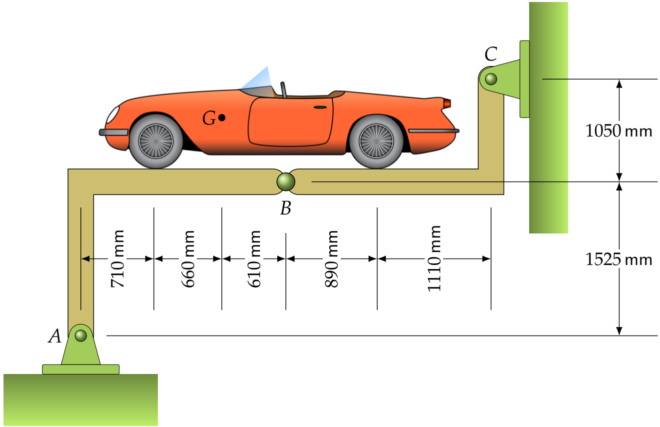
Determine the reactions at A and C.

Exercise 5



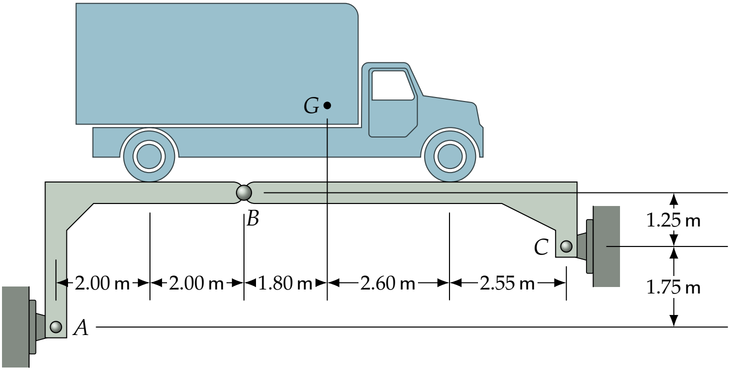
The bike and rider in front have a combined weight of 890 N. The following bike and rider have a combined weight of 970 N. Determine the reactions at A and C. (Disregard the weight of the bridge structure.)

Example 10



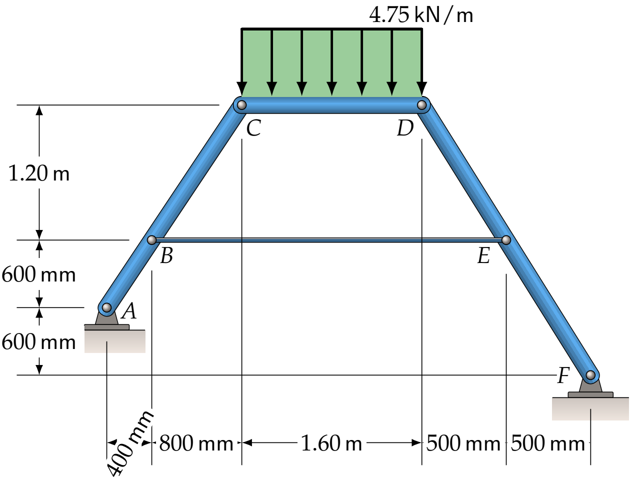
The sports car has a mass of 1100 kg and centre of gravity at G. Half of the mass is supported by the frame shown (a similar frame, hidden from view, supports the other half). All connections are pinned. Determine the reactions at A and C.

Exercise 6



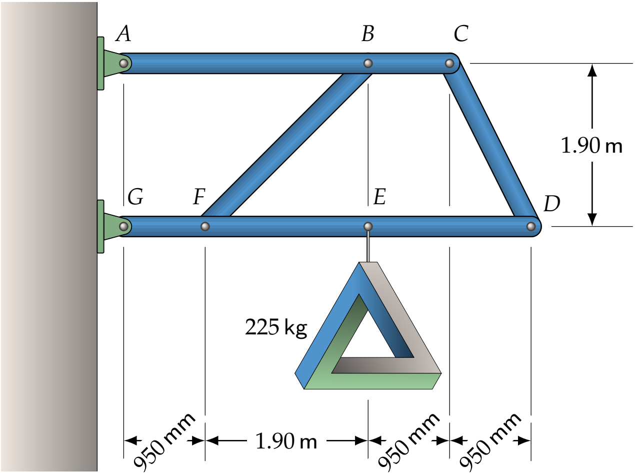
The truck has a mass of 4550 kg and centre of gravity at G. Half of the mass is supported by the frame shown (a similar frame, hidden from view, supports the other half). All connections are pinned. Determine the reactions at A and C.

Example 11



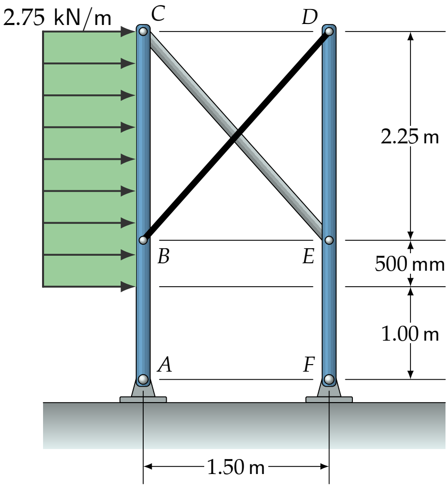
Determine the reactions at A and F, and the tension in cable BE.

Example 12



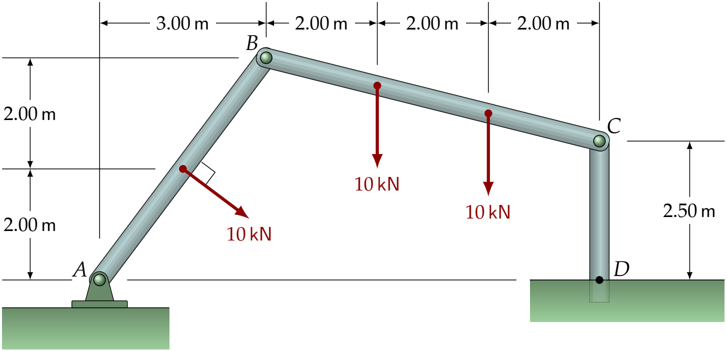
Determine the reactions at A and G.

Exercise 7



Find the forces in cable BD and strut CE, and the reactions at A and F.

Example 13



Find the reactions at A and D.