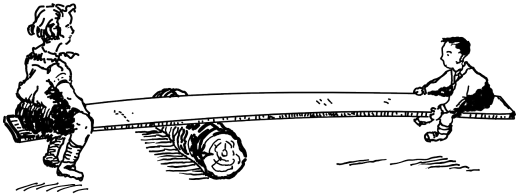
05 Moments and Couples

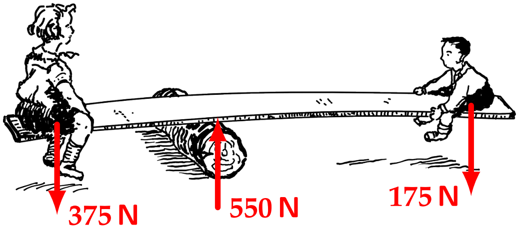
Engineering Statics

Updated on: September 22, 2025

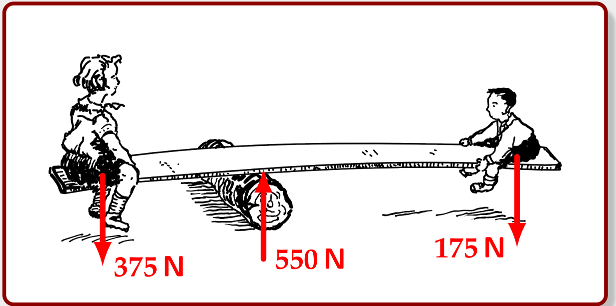
- So far, we have concerned ourselves with forces acting at a single point (or particle)
- We now start to look at non-concurrent forces, where several forces act upon a body but at different locations.



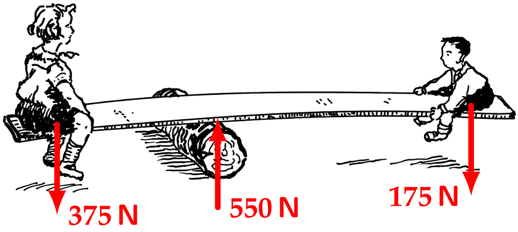
- Consider the forces acting on the beam:
- How can the teeter-totter balance when the force due to Wendy's weight is more than twice that of her younger brother?
- As children, we all figured out that the solution involved the heavier person sitting closer to the fulcrum (the balance point); this was one of our first introdutions to **moments**.



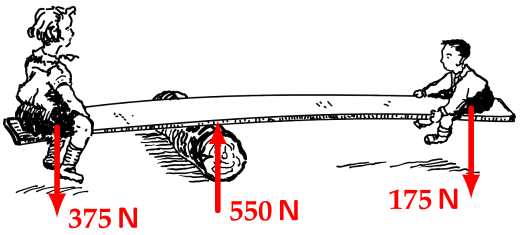
- A moment is a measure of a tendency to rotate. (It is the same as torque.)
- The teeter-totter will balance (that is, it is equilibrium) when counterclockwise moment due to Wendy equals the clockwise moment due to her little brother.
- That is, when the tendency to rotate in the counter-clockwise direction is equal to the tendency to rotate in the clockwise direction.



- Moments, then, are related to force and to distance!
- In general, when a force is applied to a structural body, there is a tendency for that body to rotate.

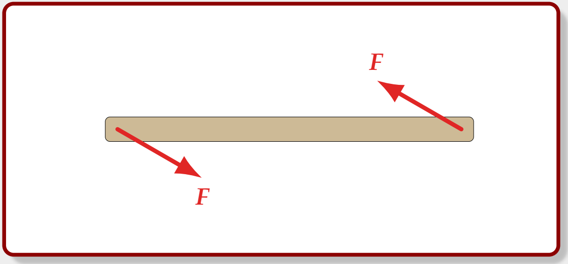


- Moments, then, are related to force and to distance!
- In general, when a force is applied to a structural body, there is a tendency for that body to rotate.
- **Important**: We assume structural bodies are rigid – they do not deform under load. Also, unless otherwise stated, structural bodies are assumed to have negligible weight compared to the forces applied to them.



Why Two Equations Are Not Enough...

- Consider a simple body, subject to two forces, equal in magnitude and opposite in direction:



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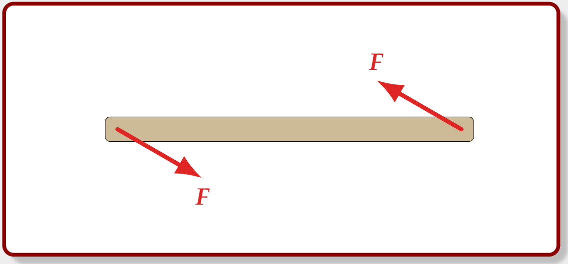

- $\Sigma F_x = 0$ and $\Sigma F_y = 0$, but, clearly, this body is not in equilibrium; it will rotate in a counter-clockwise direction.

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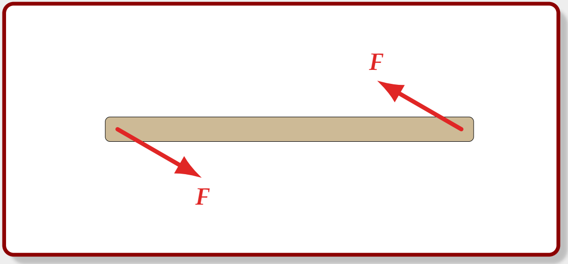
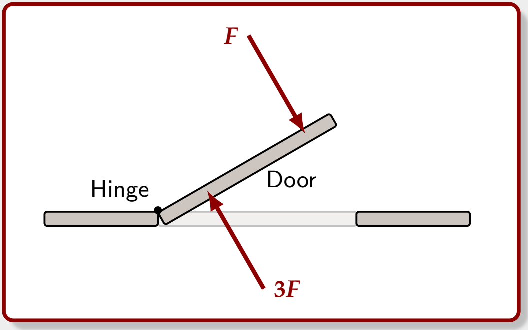


- $\Sigma F_x = 0$ and $\Sigma F_y = 0$, but, clearly, this body is not in equilibrium; it will rotate in a counter-clockwise direction.
- To consider equilibrium for non-concurrent forces, **moments** must be taken in to account.

Moments



Another lesson we learned as we were growing up was that is was easier to open a door by pushing or pulling on the handle that was further from the door hinges than it was to push close to the hinges.

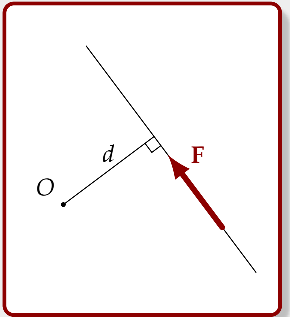
Moments

Definition of a Moment

The **moment** M_O of a force F about a point O is given by:

$$\boldsymbol{M = Fd}$$

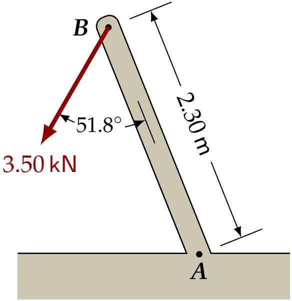
where d is the **perpendicular** distance from O to the line of action of the force F. (d is known as the **moment-arm**.)



Moments are a product of force and distance so units are $\mathrm{N \cdot mm}$, $\mathrm{N \cdot m}$, $\mathrm{kN \cdot m}$, $\mathrm{ft \cdot lb}$, $\mathrm{in \cdot lb}, \ldots$

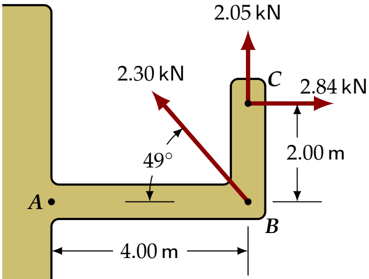
Sign Convention:
If the tendency to rotate about O is counter-clockwise, the moment is **positive**.
If clockwise, the moment is **negative**.

Example 1



Determine the moment, M_A, of the 3.5 kN force applied at B, about the point A.

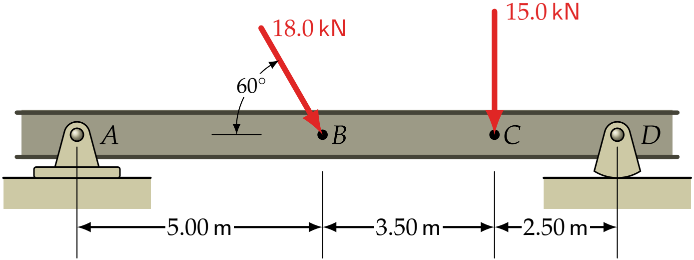
Example 2



Determine the sum of the moments of the forces, acting at B and C, about the point A.

Also, sum the moments of the forces about the point B.

Exercise 1



Rigid beam $ABCD$ is supported at A and at D, and is subjected to the two forces shown at B and C.

Determine the value of the reaction at D, R_D, if the sum of the moments about A, ΣM_A, is zero and the reaction at D is vertically upwards.

The Principle of Moments

The Principle of Moments, often referred to as Varignon's Theorem, is stated below:

Varignon's Theorem

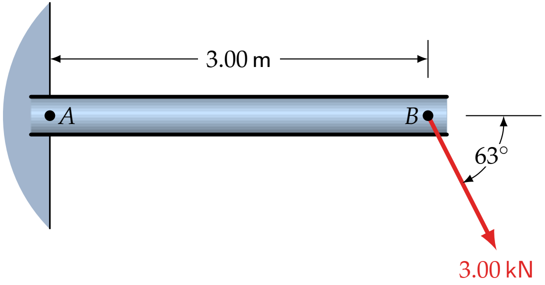
The moment of a force about a point is equal to the sum of the moments of the components of the force about the same point.

In many cases, it is more convenient to sum the moments of a forces components than it is to use the formula directly.

There are two primary ways we use this result:

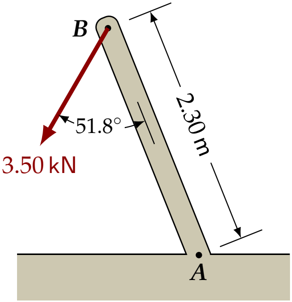
- Resolve the force into horizontal and vertical components
- Resolve the force into orthogonal (perpendicular to each other) components, with the line of action of one component passing through the point about which we are taking moments – so that its moment is zero ($d = 0$).

Example 3



Determine the moment about A of the force applied to B by resolving the force at B into horizontal and vertical components.

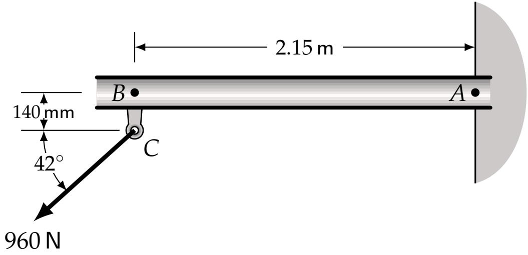
Example 4



(Revisiting *Example 1*.) Notice that we cannot calculate the moment about A of the force acting on B using horizontal and vertical components because, without the angle at which AB is inclined, we cannot determine the moment arm, d, for either the horizontal or the vertical component.

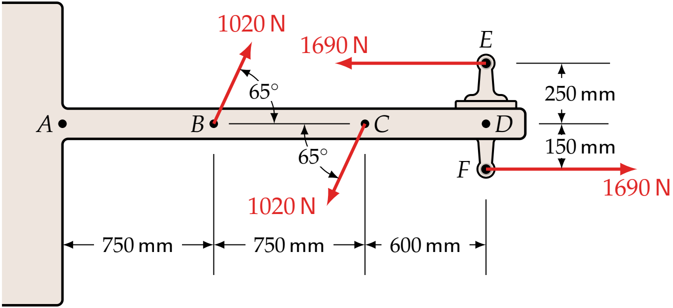
Determine the moment about A of the force at B by resolving the force into components parallel to and perpendicular to AB.

Exercise 2



Determine the moment about A of the force acting at C.
Then determine the moment about B for the same force.

Exercise 3



Determine the sum of the moments about A, about C and about F, of the forces shown.

Moments of Force Couples

- In the previous exercise, we found that the moments were the same about any of the points used.
- Also, the forces are different: there is an equal and opposite pair with magnitude 1690 N and an equal and opposite pair with magnitude 1020 N.
- Such pairs of forces, with parallel lines of action, equal magnitude and opposite sense are called force **couples**.
- The resultant force of a couple is zero: $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
- The **moment of a couple**, irrespective of any point, has magnitude

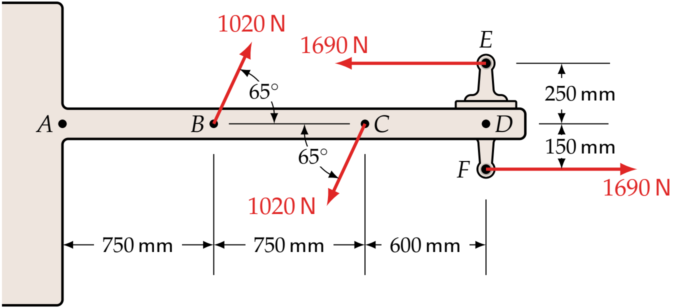
A Couple Defined:

$$M = F \cdot d$$

where d is the distance between the two lines of action.

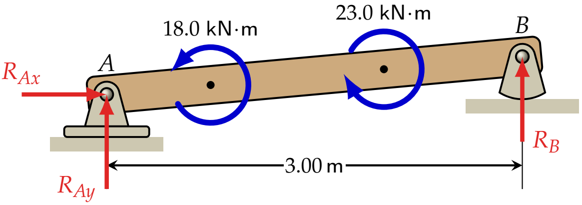
- Clockwise couples are considered negative, counter-clockwise couples are considered positive.

Example 5



Determine ΣM, the sum of the moments, of the couples shown.

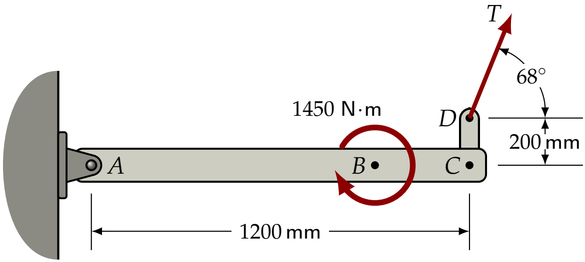
Example 6



Considering beam AB:

1. Determine the magnitude of R_D if $\Sigma M_A = 0$.
2. Determine the magnitude of R_{Ay} if $\Sigma F_y = 0$.
3. Determine the magnitude of R_{Ax} if $\Sigma F_x = 0$.

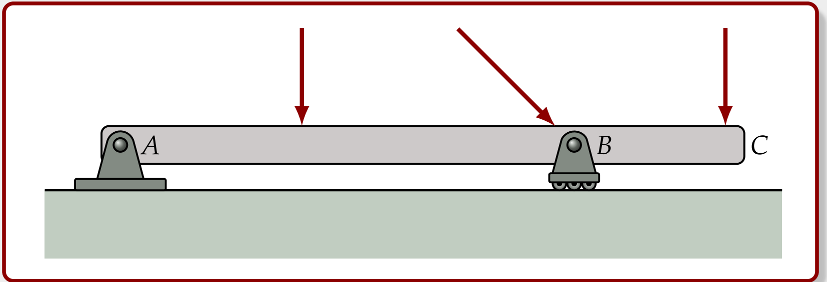
Exercise 4



$ABCD$ has a negative couple applied at B and is supported by a cable at D.

1. Determine the magnitude of T if $\Sigma M_A = 0$.
2. Determine the reaction at A if $\Sigma F_x = \Sigma F_y = 0$.

Distributed Loads

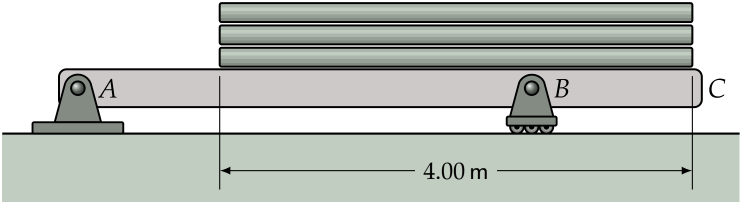


- Consider a rigid beam ABC with individual loads concentrated at separate locations. We have found the moments of forces like these.

Distributed Loads

- Suppose, instead, that three W150 X 13 beams are placed on ABC.
- If all the beams are rigid (as we assume in statics), then the weight of the load is distributed uniformly to ABC along the length of the three W150 X 13 beams
- This kind of load is known as a **uniformly distributed load** (UDL).

Distributed Loads

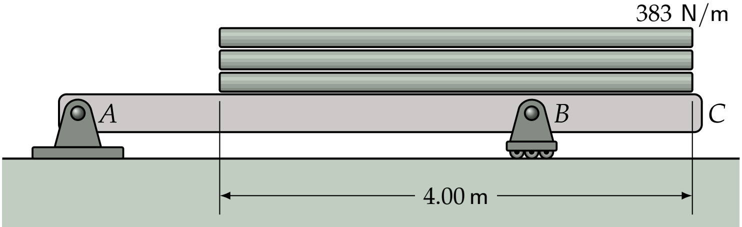


- Now, assume the three beams are each 4.00 m in length.
  The total mass, m, and weight, W, of the three beams is given by:

$$m = 3 \text{ @ } 4.00 \text{ m} \times 13\,\text{kg} = 156\,\text{kg}$$

$$W = 156\,\text{kg} \times 9.81\,\text{m/s}^2 = 1530 \text{ N}$$

Distributed Loads



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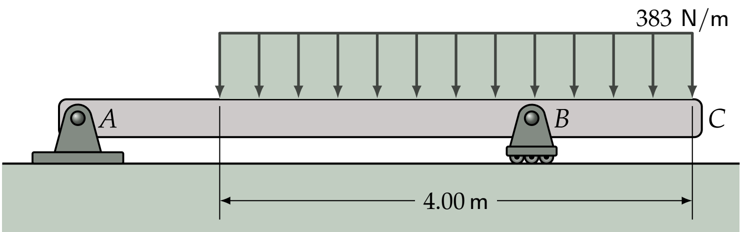
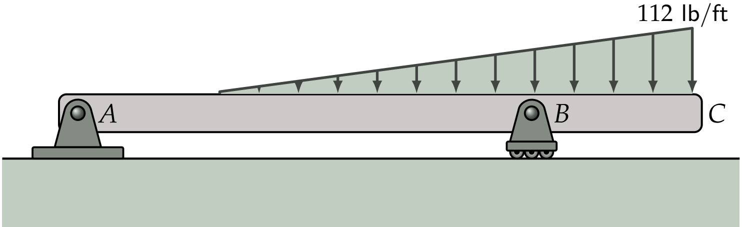
- This force is spread out uniformly over 4.00 m so the distributed load is 382.50 N/m.
- Note that distributed loads have units force/length: **N/m**, **lb/ft**.

Distributed Loads



- Uniformly distributed loads are drawn as shown.

Distributed Loads



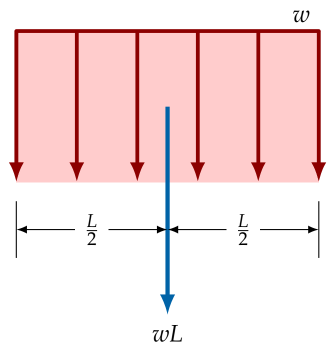
- Distributed loads need not be uniform. They can be **uniformly varying loads** (UVL).
- Note that uniformly varying loads vary at a constant rate so they have a triangular shape, with a constant slope.

  (Distributed loads may also defined as $f(x)$, a function of x where x is the distance along the load, and may follow fairly complex curves that require integration to solve: we shall not cover these in this course.)

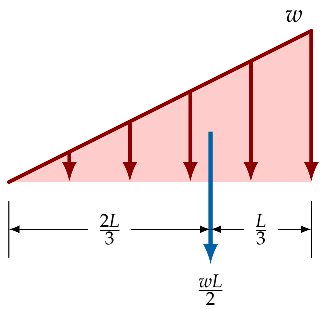
Magnitude and Resultant of Distributed Loads

In both cases, the magnitude of the resultant force is given by the area of the rectangle or triangle, and the resultant acts through the centroid of the area.

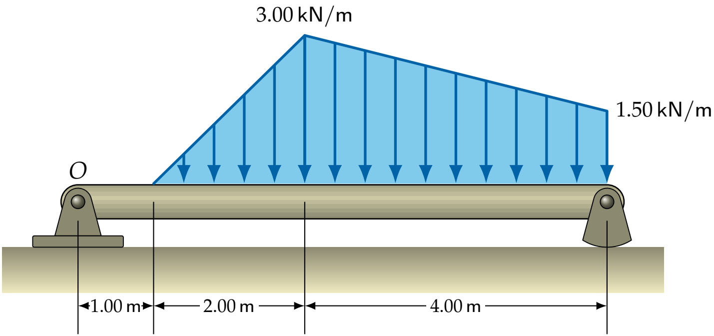
Uniformly Distributed Load



Uniformly Varying Load

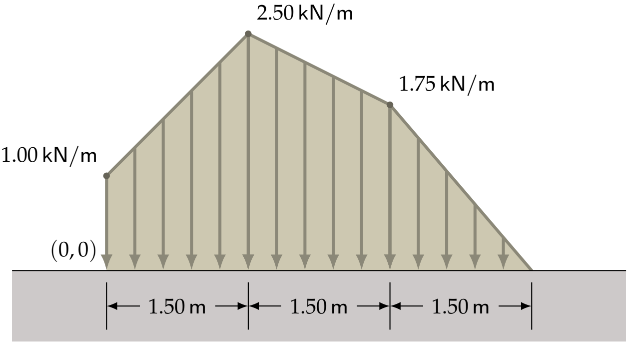


Example 7



Determine the moment of the distributed load about O.

Exercise 5



Determine the moment of the distributed load about $(0, 0)$.