04 Centroids

Engineering Statics

Updated on: September 17, 2025

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- Why learn it now?
  - Centroids are used in the analysis of **distributed loads**, an important part of statics (and future courses!).
  - The forces we have considered so far have been **concentrated loads**, meaning that each force acts at (is concentrated at) a single point. Distributed loads are spread out, e.g. snow on a roof, water against side of a swimming pool, wind on the wall of a building.

The Centre of Gravity

- A body is made up of infinitely many small elements, each having a weight (a downward force due to gravity).
- The resultant of these parallel forces is also downward and the line of action of this resultant goes through a point called the **centre of gravity**.
- The centre of gravity may be considered as the 'balance point' for the body. It is the point where the total weight of the body acts.

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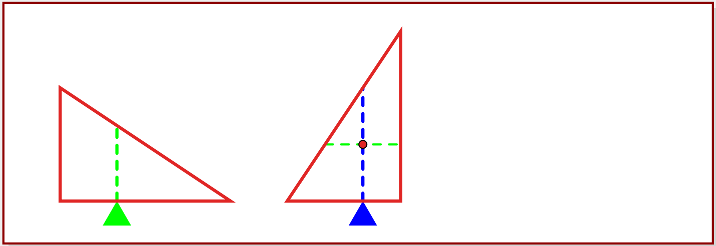

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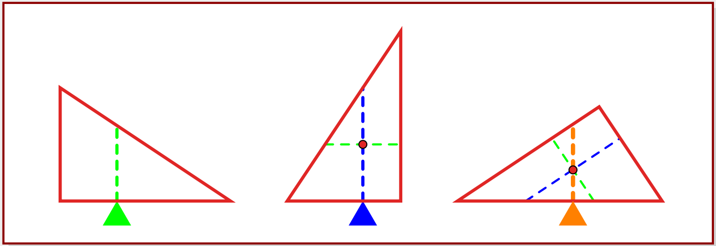
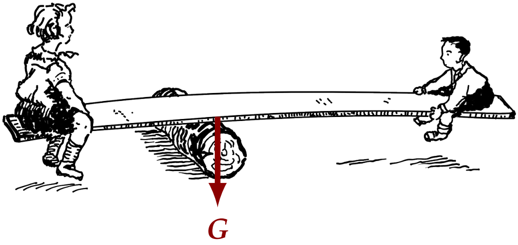
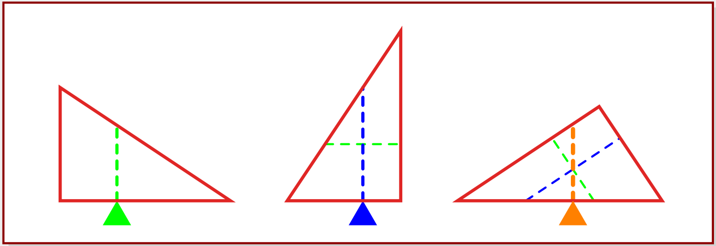


- If the see-saw is in equilibrium (i.e., if it is static or 'balanced'), then the line of action of centre of gravity of the board and the two children passes through the supporting log.



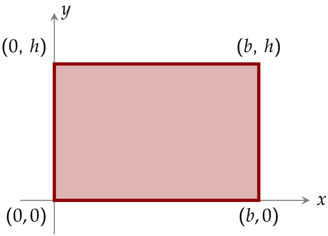
The Centroid



- The resultant forces lines of action for the triangle shown assume a constant density of material.
- If part of the triangle was constructed from steel and the rest from wood, the balance point might be very different.
- But, if the body is of a uniform material, then the centre of gravity is at the geometric centre of the triangle shape. This geometric centre is called the **centroid** of the area.
  (You can think of it as the centre of gravity of the area.)
- In civil engineering, we are mainly concerned with the centroid.

The centroids of simple shapes are well-known. We shall just assume the results for these simple shapes: rectangles, triangles, circles, semi-circles and quarter circles

The Centroid of a Rectangle



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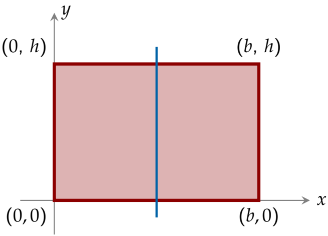

- The rectangle has a vertical axis of symmetry, along the line $x = b/2$

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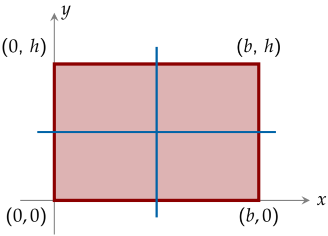

- The rectangle has a vertical axis of symmetry, along the line $x = b/2$

  And a horizontal axis of symmetry along the line $y = h/2$.

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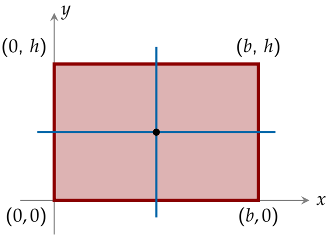
The Centroid of a Rectangle



- The rectangle has a vertical axis of symmetry, along the line $x = b/2$

  And a horizontal axis of symmetry along the line $y = h/2$.
- If an area has an axis of symmetry, then the centroid will lie on that axis. In this case, the centroid must lie on both the vertical and the horizontal axes of symmetry.

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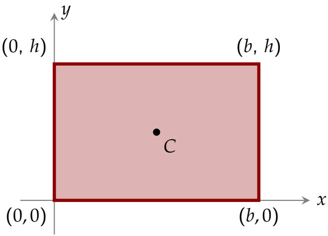


- The rectangle has a vertical axis of symmetry, along the line $x = b/2$

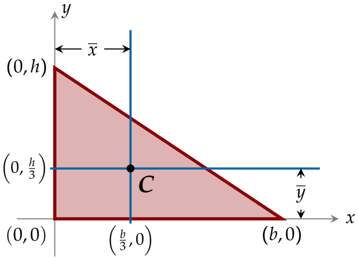
  And a horizontal axis of symmetry along the line $y = h/2$.
- If an area has an axis of symmetry, then the centroid will lie on that axis. In this case, the centroid must lie on both the vertical and the horizontal axes of symmetry.
- The centroid of the rectangle is at

  $$(\bar{x}, \bar{y}) = \left(\frac{b}{2}, \frac{h}{2}\right)$$

  where $\bar{x}$ and $\bar{y}$ designate the x and y coordinates of the centroid.

We focus on right triangles since they generally satisfy our needs.

The Centroid of a Right Triangle

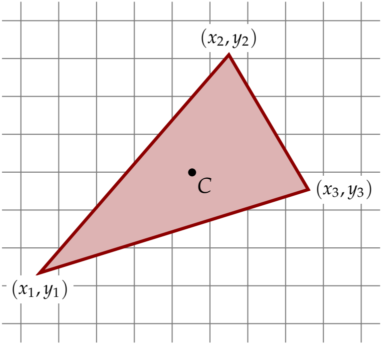


- The centroid, C, of the triangle is located at

$$(\overline{x},\overline{y}) = \left(\frac{b}{3},\frac{h}{3}\right)$$

The centroid of **any** triangle is given by the average values of its vertices.

The Centroid of any Triangle



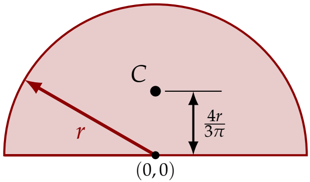
The centroid, C, of the triangle is located at

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

The centroid of a circle is at the circle's centre
(the intersection of all axes of symmetry).

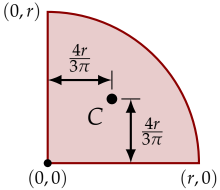
The semi-circle and the quarter-circle are less obvious.

Centroid of a Semi-Circle



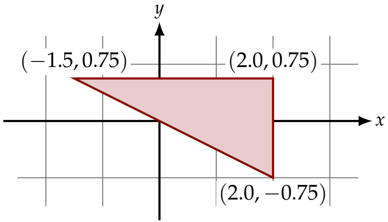
$$(\bar{x}, \bar{y}) = \left(0, \frac{4r}{3\pi}\right)$$

Centroid of a Quarter-Circle



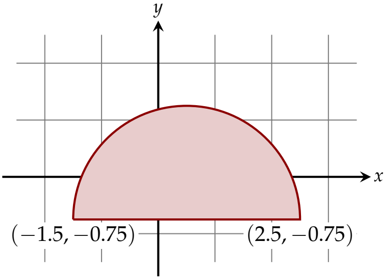
$$(\bar{x}, \bar{y}) = \left(\frac{4r}{3\pi}, \frac{4r}{3\pi}\right)$$

Exercise 1



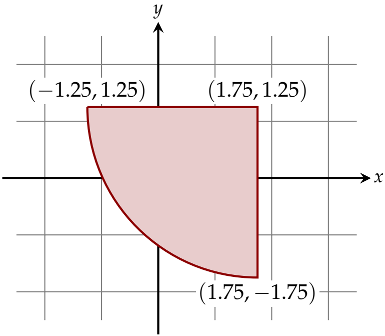
Determine the location of the centroid of the triangle shown.

Exercise 2



Determine the location of the centroid of the semi-circle shown.

Exercise 3



Determine the location of the centroid of the quarter-circle shown.

Composite Shapes

Typically we are concerned with the properties of more complex shapes, composed of a number of simple shapes or standard structural cross-sections (such as wide-flange beams, channels, angles or plates).

Whether we are dealing with simple geometrical shapes or with standard sections, the centroid of the composite shape is given:

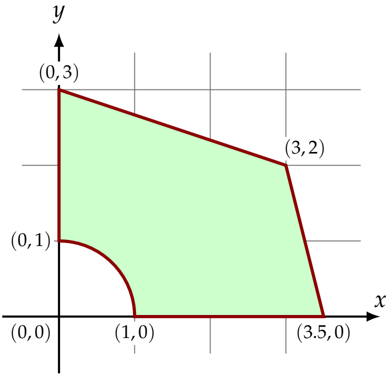
Centroid of a Composite Shape

$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \dots}{A_1 + A_2 + A_3 + \dots}$$

$$\bar{y} = \frac{\Sigma A y}{\Sigma A} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \dots}{A_1 + A_2 + A_3 + \dots}$$

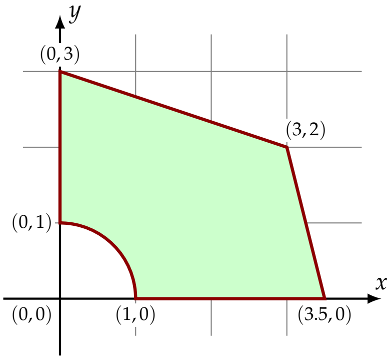
where the A_i are the areas of the simple shapes making up the composite shape and the x_i and y_i are the $\bar{x}$ and the $\bar{y}$ of the simple shapes.

Example 1



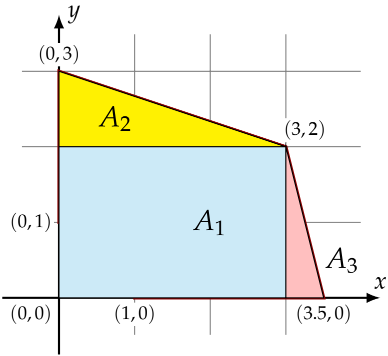
Find the location of the centroid, C, relative to the coordinate origin.

Example 1: Solution



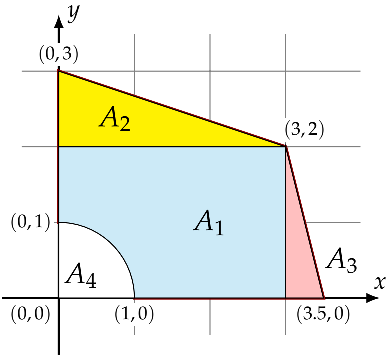
1. Divide the composite shape into simple shapes:

Example 1: Solution



1. Divide the composite shape into simple shapes:
   - Notice that the 'missing' quarter-circle is covered up by rectangle A_1.

Example 1: Solution



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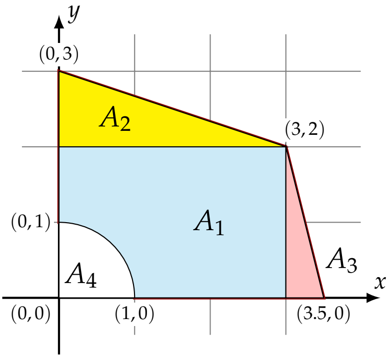
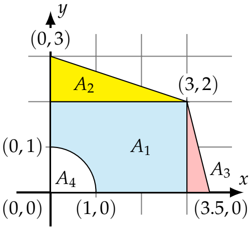


1. Divide the composite shape into simple shapes:
   - Notice that the 'missing' quarter-circle is covered up by rectangle A_1.
   - Draw in the quater-circle; we shall treat this area as negative
2. Apply the formulæ for $\overline{x}$ and $\overline{y}$.
3. For complicated shapes, you may want to arrange your data in a table.

Example 1: Solution



$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{6(1.5) + 1.5(1) + 0.5(3.1667) - \frac{\pi}{4}\left(\frac{4\times 1}{3\pi}\right)}{6 + 1.5 + 0.5 - \frac{\pi}{4}}$$

$$= \frac{11.750}{7.2146} = 1.6286$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{6(1) + 1.5(2.3333) + 0.5(0.66667) - \frac{\pi}{4}\left(\frac{4\times 1}{3\pi}\right)}{7.2146}$$

$$= \frac{9.5000}{7.2146} = 1.3168$$

The centroid is at the location $(\bar{x}, \bar{y}) = (1.63, 1.32)$.

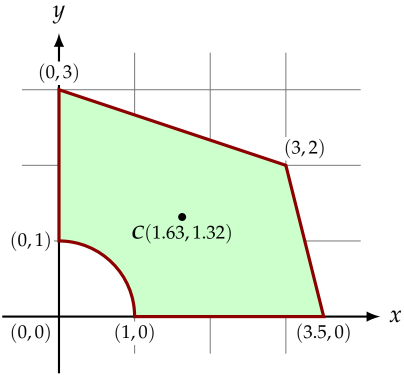
Composite Shapes

Example 1: Solution Using Table

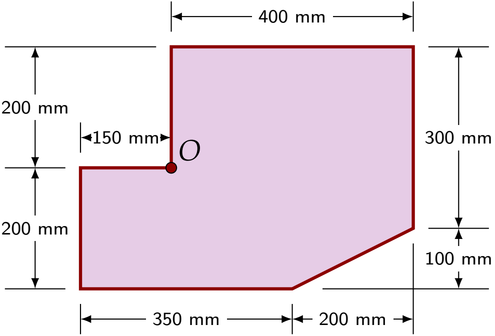
Shape	Area	x_i	y_i	$A_i x_i$	$A_i y_i$
A_1	6.0000	1.5000	1.0000	9.0000	6.0000
A_2	1.5000	1.0000	2.3333	1.5000	3.5000
A_3	0.50000	3.1667	0.66667	1.5834	0.33333
A_4	$-\frac{\pi}{4}$	$\frac{4(1)}{3\pi}$	$\frac{4(1)}{3\pi}$	-0.33333	-0.33333
Σ	7.2146			11.750	9.5000

$$(\bar{x}, \bar{y}) = \left(\frac{\Sigma A_i x_i}{A}, \frac{\Sigma A_i y_i}{A} \right) = \left(\frac{11.750}{7.2146}, \frac{9.5000}{7.2146} \right) = (1.63, 1.32)$$

Example 1: The Answer

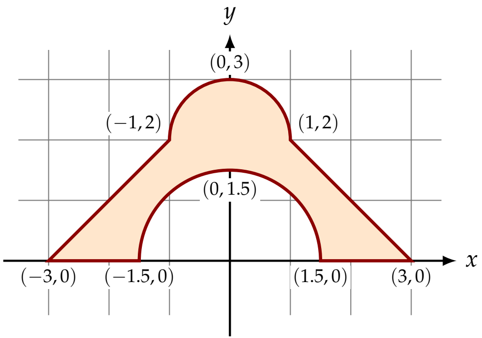


Exercise 4



Find the location of the centroid, C, relative to the point O.

Exercise 5



Find the location of the centroid, C, relative to the coordinate origin.

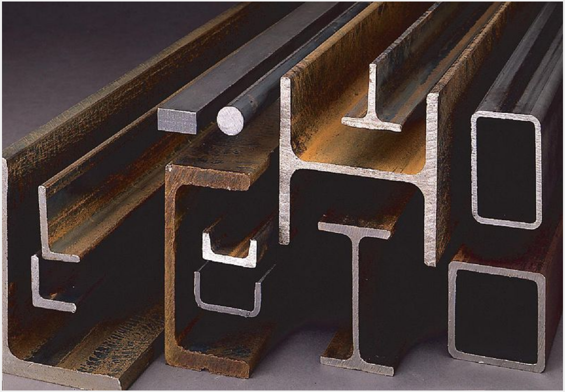
Structural Steel

This modern Calgary high-rise contains a lot of steel...

Structural Steel

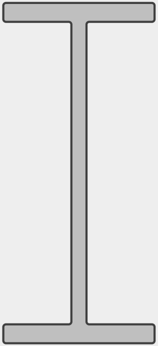
Just some of the steel that went into the highrise...

Structural Steel



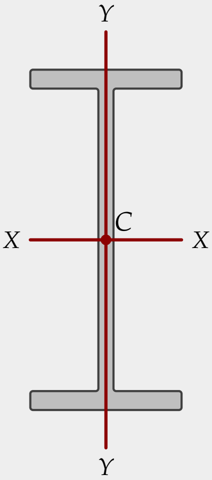
- There are many structural steel sections used in construction.
- Details of their physical dimensions and structural properties are widely available in tables.
- Often, various sections are welded together to suit a particular need.

Wide-flanged I-beams



- A common beam used in construction has this characteristic I-shaped cross-section.
- This is called a wide-flanged I-beam due to the large horizontal flanges top and bottom.
- It has three parts: top and bottom horizontal **flanges** and a vertical **web**.
- These beams are designated in the form W depth x mass per unit length.
- A W460x82 beam has a wide flange (hence the W), a nominal depth of 460mm and a mass of 82 kg per metre.

Wide-flanged I-beams



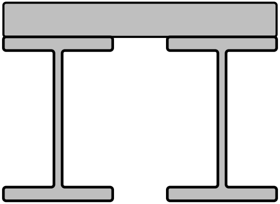
- The beam has a vertical axis of symmetry so the centroid must be on that line.
- The beam also has a horizontal axis of symmetry so the centroid must be on that line, too.
- The centroid of the cross-sectional area of the beam is easily found at the intersection of the two axes of symmetry.

Structural Steel

One of these beams may support your floor system at home...

...or the ceiling above my office...

Example 2



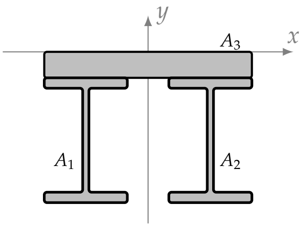
Two W250X115 steel beams have a steel plate welded on top of them, as shown.
The dimensions of the plate are 650 mm x 60 mm.

How far below the top of the built-up section is the centroidal x-axis?

Properties for W250X115:
Area $= 14600\,\text{mm}^2$
Depth $= 269\,\text{mm}$

(Beam tables at beamdimensions.com)

Example 2: Solution

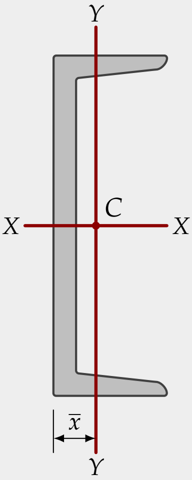


Shape	A_i (mm^2)	y_i (mm)	$A_i y_i$ (mm^3)
A_1	14600	−134.5	−1963700
A_2	14600	−134.5	−1963700
A_3	39000	−30	−1170000
Σ	68200		−5097400

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = -74.742 \text{ mm}$$

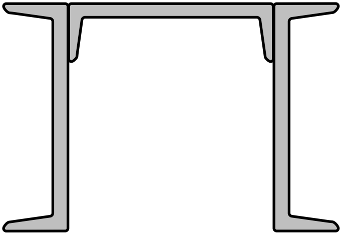
The centroidal x-axis of the built-up shape is 74.7 mm below the top of the plate.

Channel Section



- The channel section has a horizontal axis of symmetry $(X - X)$ but no vertical axis of symmetry.
- The centroidal y-axis is specified by $\overline{x}$ (See the table A.5 in the course text for a particular section, sizes and corresponding properties.)

Example 3



Two C180X22 and one C150X19 are welded together as shown.

How far below the top of the built-up section is the centroidal x-axis?

Properties for C180X22:

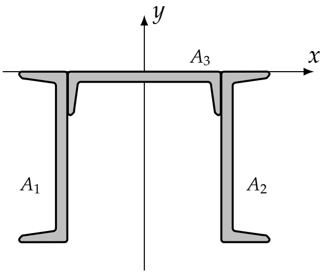
Area $= 2790\,\text{mm}^2$
Depth $= 178\,\text{mm}$
$\bar{x} = 13.5\,\text{mm}$

Properties for C150X19:

Area $= 2470\,\text{mm}^2$
Depth $= 152\,\text{mm}$
$\bar{x} = 12.9\,\text{mm}$

(Beam tables at beamdimensions.com)

Example 3: Solution

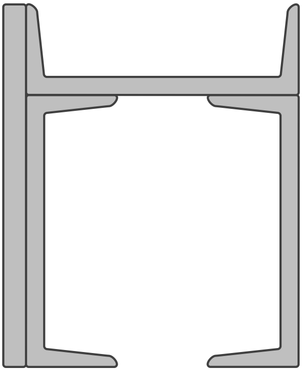


Shape	A_i (mm^2)	y_i (mm)	$A_i y_i$ (mm^3)
A_1	2790	−89	−248310
A_2	2790	−89	−248310
A_3	2470	−12.9	−31863
Σ	8050		−528483

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = -65.650 \text{ mm}$$

The centroidal x-axis of the built-up shape is 65.7 mm below the top of the plate.

Exercise 6



Three C130X13 and a steel plate (15mm $\times$ 174mm) are welded together.

Determine the location of the centroid, relative to the bottom left hand corner of the composite area.

Properties for C130X13:

Area $= 1700\,\text{mm}^2$
Depth $= 127\,\text{mm}$
$\overline{x} = 12\,\text{mm}$

(Beam tables at beamdimensions.com)