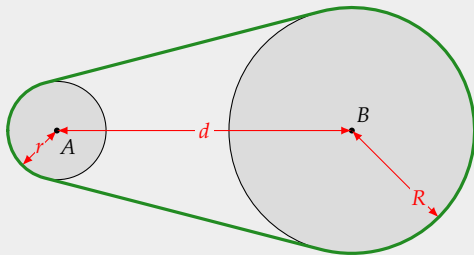


# *Miscellaneous Nerdery*

Updated on: September 9, 2025

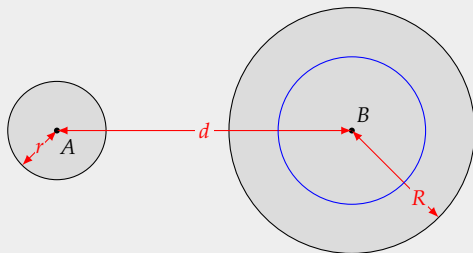
## Geometry :: Belt-Length



Two pulleys, centred at  $A$  and  $B$ , have radii  $r$  and  $R$ . The distance from  $A$  to  $B$  is  $d$ .

Determine the length of the belt required to go round both pulleys.

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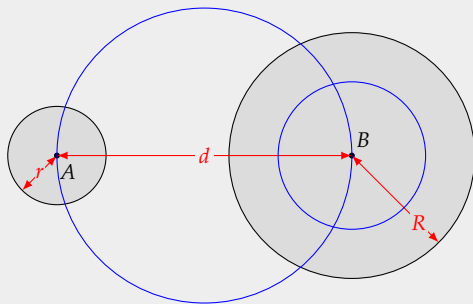


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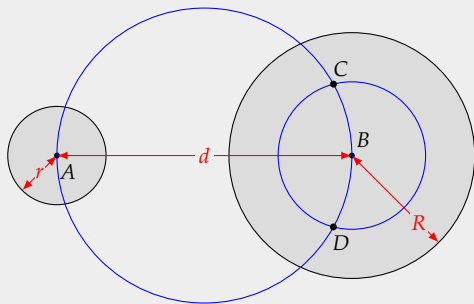


1. Construct a circle, diameter  $R-r$ , centred at  $B$ .
2. Construct a circle with diameter  $AB$ .

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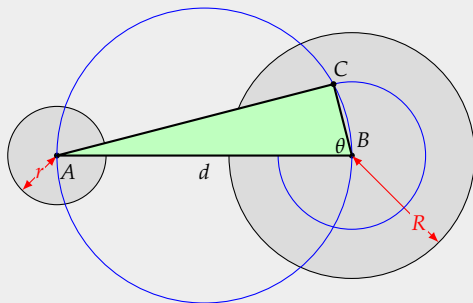


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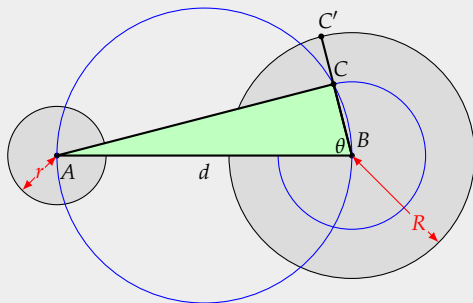
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4. Consider  $\triangle ABC$ :  $\angle ACB = 90^\circ$  since it is an angle inscribed in a semicircle. Then:

$$AC = \sqrt{d^2 - (R-r)^2}$$

$$\theta = \sin^{-1} \left( \frac{\sqrt{d^2 - (R-r)^2}}{d} \right)$$

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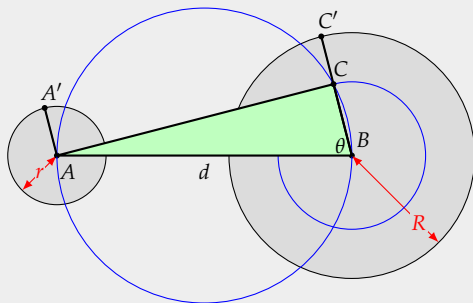
5. Extend line  $BC$  to  $C'$  on the circumference of pulley  $B$ .  $CC'$  has length  $r$ .

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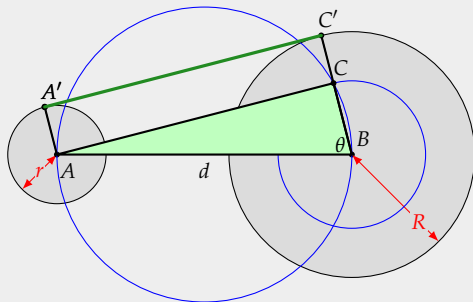
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7. Draw  $A'C'$ :  $A'C'CA$  is a rectangle so

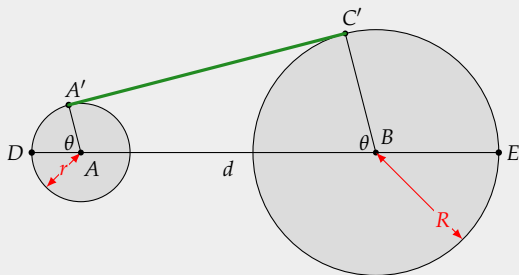
$$A'C' = AC = \sqrt{d^2 - (R - r)^2}$$

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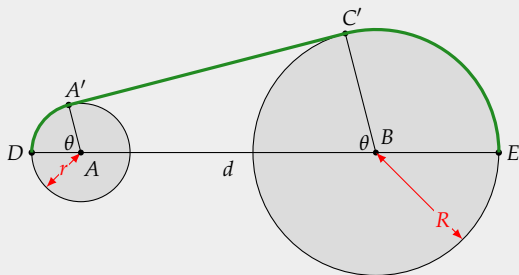
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8.  $A'C'$  is the (top) part of the belt that is tangential to the pulleys at  $A'$  and  $C'$ . We now need to find the arc-lengths from  $D$  to  $A'$  and from  $C'$  to  $E$ .

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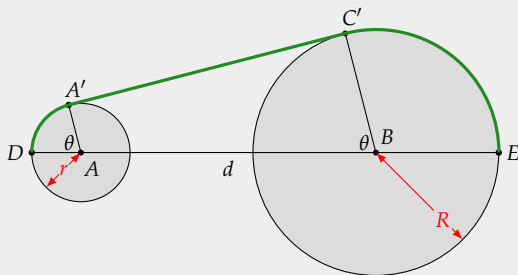
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9. The angles ( $\theta$  and  $\pi - \theta$ ) that these arcs subtend at the pulley centres, with the radius of each pulley, are used to determine the arc-lengths ( $\theta$  in radians):

$$DA' = r\theta \text{ and } C'E = R(\pi - \theta)$$

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Determine the length of the belt required to go round both pulleys.

10. Belt-length:

$$\begin{aligned}
 &= 2 (DA' + A'C' + C'E) \\
 &= 2 \left( r\theta + \sqrt{d^2 - (R - r)^2} + R(\pi - \theta) \right)
 \end{aligned}$$

where

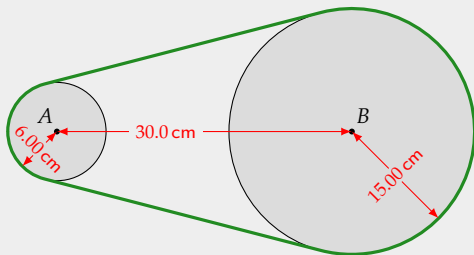
$$\theta = \sin^{-1} \left( \frac{\sqrt{d^2 - (R - r)^2}}{d} \right)$$

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Example:

$$\theta = \sin^{-1} \left( \frac{\sqrt{d^2 - (R - r)^2}}{d} \right) = \sin^{-1} \left( \frac{\sqrt{6.00^2 - 1.50^2}}{6.00} \right) = 1.3181 \text{ (radians)}$$

$$\text{B-L} = 2 \left( 6.00 \times 1.3181 + \sqrt{6.00^2 - 1.50^2} + 15.00 \times (\pi - 1.3181) \right) = 82.141$$

The belt length is 82.1 cm.